



# Predictability of thermoelectric figure of merit for the single crystal from first principles

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## ABSTRACT

The first-principles calculations have been widely used to predict the thermoelectric figure of merit ( $ZT$ ) for the single crystal, but such precise calculations are confined to phonon-limited intrinsic system, and the defects present in practical samples cannot be easily modeled. So, a question arises as to whether the phonon-limited  $ZT$  can represent the thermoelectric potential of the material. In this work, we demonstrate that the exclusion of defect scattering could provide  $ZT$  close to the actual values in single crystal by analyses of the  $ZT$  involved parameters. This is also illustrated in the case of SnSe by performing first-principles calculations of phonon-limited  $ZT$  compared with available experimental data. The calculated  $ZT$  can reproduce the measured values, indicating that the calculations possess predictive capability. On the basis, strong and anisotropic thermoelectric enhancement of SnSe induced by pressure is found. The pressure significantly increases the thermoelectric power factor of  $n$ -type SnSe along its  $a$  crystallographic axis and that of  $p$ -type SnSe along the  $b$  axis, while the increase of lattice thermal conductivity is relatively small. As a result, a large enhancement of  $ZT$  is obtained. By applying pressure up to 3.2 GPa, at room temperature, the  $a$ -axis  $ZT$  of  $n$ -type SnSe is as high as 1.2, while the  $b$ -axis  $ZT$  of  $p$ -type SnSe reaches up to 0.8, which are more than twice larger than the values at ambient pressure. This study provides strong confidence and support for the application of first-principles calculations to search high-performance thermoelectric materials and thus would be an important reference for future researches.

## 1. Introduction

Thermoelectric technology enables direct and reversible conversion between heat and electricity, which provides a promising solution in waste heat recovery and electronic refrigeration. The key to the popularization of practical applications lies in the energy conversion efficiency of thermoelectric materials, which is determined by the dimensionless figure of merit  $ZT = \alpha^2 \sigma T / (\kappa_e + \kappa_l)$ , where  $\alpha$  is the Seebeck coefficient,  $\sigma$  is the electrical conductivity,  $T$  is the absolute temperature,  $\kappa_e$  and  $\kappa_l$  are the thermal conductivities contributed by electron transport and phonon transport, respectively. Finding materials with high  $ZT$  is the essential demand for the enhancement of thermoelectric performance. Theoretical prediction is an efficient way to screen out high  $ZT$  candidates from thousands of systems at a small cost as compared to experiments. The primary task is the accurate calculations of  $ZT$  involved transport properties. In pristine single crystals, the intrinsic  $\kappa_l$  and  $\{\alpha, \sigma, \kappa_e\}$  are limited by the phonon-phonon and

electron-phonon interactions, respectively, which can be extracted from density functional theory (DFT) and density functional perturbation theory (DFPT) calculations. Combined with Boltzmann transport equations (BTE), the phonon and electron transport properties can be obtained in the parameter-free framework at first-principles accuracy. The first-principles calculations of intrinsic  $\kappa_l$  have been well developed in the past decades [1–3] and have been widely used to search thermoelectric candidates with low- $\kappa_l$  following the phonon-glass electron-crystal paradigm [4,5]. The calculations of intrinsic  $\{\alpha, \sigma, \kappa_e\}$  have also been achieved in first-principles framework recently and have been successfully applied to many materials with satisfactory results [6–9]. Therefore, it is already feasible to derive the phonon-limited  $ZT$  by predicting all relevant quantities through the first-principles calculations [10–13].

In practical applications, the single crystals generally contain intermediate or strong doping, bringing defects into crystal structures. However, modeling the defects with predictive accuracy cannot be eas-

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ily achieved for two reasons: first it is hard to acquire the knowledge of defects experimentally, and second it lacks powerful treatment of defects in calculations. Recently, the calculations of defects-included phonon transport [14,15] and electron transport [16,17] have been demonstrated with high accuracy by calculating the perturbation matrix of supercells including defects. Nonetheless, only simple types of point defects like substitutions and vacancies can be considered and the computational cost for perturbed supercell system is dramatically multiplied as compared to that of traditional treatment on primitive cell. Furthermore, single crystals are the parent materials for the thermoelectric enhancement. Therefore, the calculation without considering defects is still the priority choice in terms of rapid pre-research. As a matter of course, a concern arises about whether the phonon-limited  $ZT$  does represent the thermoelectric potential of materials.

To evidence the ability of first-principles calculations in  $ZT$  prediction for single crystals, SnSe is a rather ideal alternative for comparing calculations and experiments due to its numerous measurements of  $n$ - and  $p$ -type doped systems. SnSe has anisotropic layered crystal structure and undergoes continuous phase transition from  $Pnma$  to  $Cmcm$  phase starting around 600 K to the critical temperature of 800 K [18–20]. The remarkable thermoelectric properties of SnSe were first noticed by Zhao et al. [18], reporting an unprecedented high  $ZT$  of  $2.6 \pm 0.3$  at 923 K in the unintentionally doped  $p$ -type system along the  $b$  axis direction due to its intrinsically ultralow lattice thermal conductivity. The authors later realized heavy hole doping in single crystals of SnSe successfully by introducing sodium dopants, and the  $ZT$  was vastly increased to 0.7 at 300 K while the maximum  $ZT$  reaches up to 2.0 at 773 K along the  $b$  axis direction owing to the enhanced electrical conductivity and Seebeck coefficient afforded by doping [21]. Subsequently,  $n$ -type SnSe was obtained via bismuth doping, achieving a  $ZT$  value of 2.2 at 733 K along the  $b$  axis direction [22]. The  $n$ -type SnSe was also synthesized through bromine doping, which generates overlapping interlayer charge density to form three-dimensional charge transport but remains two-dimensional phonon transport in the out-of-plane direction, resulting in a record  $ZT$  value of  $2.8 \pm 0.5$  at 773 K [19]. The average  $ZT$  at 300 to 773 K was further improved in chlorine-doped, lead-alloyed and copper-doped SnSe by multi-band synglisis, phonon-electron decoupling and lattice plainification [23–25].

Previously, Li et al. [12] have used first-principles methods to calculate the  $ZT$  of  $Pnma$  SnSe, taking into account the effect of point dopant scattering in phonon transport with an empirical simplified formula but not in electron transport. The obtained  $ZT$  at 300 K for  $n$ - and  $p$ -type SnSe are acceptable as compared with the experiments of Chang et al. [19] and Zhao et al. [21], respectively. In contrast to this research, we focus on the comparison between calculated  $ZT$  of defect-free crystals and experimental values at more comprehensive temperatures and carrier concentrations. Another issue to be aware of is that the thermoelectric performance of  $Pnma$  SnSe can be enhanced by pressure [26,27] due to the changes in crystal structure and thus leading to a certain influence on electron and phonon properties. Recent experiments have demonstrated that the pressure induces Lifshitz transition in  $Pnma$  SnSe and thus largely enhances the thermoelectric power factor over a large temperature range [28]. However, previous calculations adopted constant [26] and simplified empirical [27] approximations for electron-phonon scattering, which may cause large deviations from the real physical processes especially for such highly anisotropic materials. Therefore, in this work, we calculated all the  $ZT$  involved transport properties of  $Pnma$  SnSe using fully first-principles methods. It is found that the measured  $ZT$  does not deviate from the calculated values significantly, demonstrating the speculation that the calculation without including the defect scattering is able to reveal the thermoelectric promise of single crystals. On this basis, the  $ZT$  of  $Pnma$  SnSe under different pressures are investigated. By applying pressure, the  $ZT$  of  $n$ - and  $p$ -type SnSe are significantly enhanced in the  $a$ - and  $b$ -axis directions, respectively, but no obvious improvement in other directions.

## 2. Methodology

The electrical transport properties of  $\sigma$ ,  $\alpha$  and  $\kappa_e$  can be derived from the framework of BTE for electron, as [11,29]

$$\sigma = \frac{se^2}{\Omega N_{\mathbf{k}}} \sum_{n\mathbf{k}} v_{n\mathbf{k}} \mathbf{F}_{n\mathbf{k}} \left( -\frac{\partial f_{n\mathbf{k}}^0}{\partial \epsilon_{n\mathbf{k}}} \right), \quad (1)$$

$$\alpha = \frac{\sigma^{-1} se}{T \Omega N_{\mathbf{k}}} \sum_{n\mathbf{k}} (\epsilon_{n\mathbf{k}} - \epsilon_f) v_{n\mathbf{k}} \mathbf{F}_{n\mathbf{k}} \left( -\frac{\partial f_{n\mathbf{k}}^0}{\partial \epsilon_{n\mathbf{k}}} \right), \quad (2)$$

$$\kappa_e = \frac{s}{T \Omega N_{\mathbf{k}}} \sum_{n\mathbf{k}} (\epsilon_{n\mathbf{k}} - \epsilon_f)^2 v_{n\mathbf{k}} \mathbf{F}_{n\mathbf{k}} \left( -\frac{\partial f_{n\mathbf{k}}^0}{\partial \epsilon_{n\mathbf{k}}} \right) - T \sigma \alpha^2, \quad (3)$$

where  $N_{\mathbf{k}}$  is the total number of  $\mathbf{k}$  points,  $\Omega$  is the volume of unit cell,  $e$  is the electron charge,  $f_{n\mathbf{k}}^0$  is the Fermi-Dirac distribution for  $n\mathbf{k}$  state,  $v_{n\mathbf{k}}$  is the electron velocity,  $\epsilon_{n\mathbf{k}}$  is the electron energy and  $\epsilon_f$  is the chemical potential,  $s$  is the number of electrons per state.  $\mathbf{F}_{n\mathbf{k}}$  is obtained from iterative solution of electron BTE,

$$\mathbf{F}_{n\mathbf{k}}^{i+1} = \tau_{n\mathbf{k}} v_{n\mathbf{k}} + \frac{\tau_{n\mathbf{k}}}{N_{\mathbf{q}}} \sum_{m\mathbf{q}} W_{n\mathbf{k},m\mathbf{k}+\mathbf{q}}^{m\mathbf{q}} \mathbf{F}_{m\mathbf{k}+\mathbf{q}}^i, \quad (4)$$

starting from  $\mathbf{F}_{n\mathbf{k}} = v_{n\mathbf{k}} \cdot \tau_{n\mathbf{k}}$  with  $\tau_{n\mathbf{k}}$  being the relaxation time.  $N_{\mathbf{q}}$  is the total number of  $\mathbf{q}$  points,  $W_{n\mathbf{k},m\mathbf{k}+\mathbf{q}}^{m\mathbf{q}}$  is the phonon-limited transition probability via absorption and emission processes [30]. Analogously, the  $\kappa_l$  also can be obtained from the BTE for phonon, written by

$$\kappa_p = \frac{1}{\Omega N_{\mathbf{q}}} \sum_{\mathbf{q}p} \hbar \omega_{\mathbf{q}p} v_{\mathbf{q}p} \mathbf{F}_{\mathbf{q}p} \left( \frac{\partial n_{\mathbf{q}p}^0}{\partial T} \right), \quad (5)$$

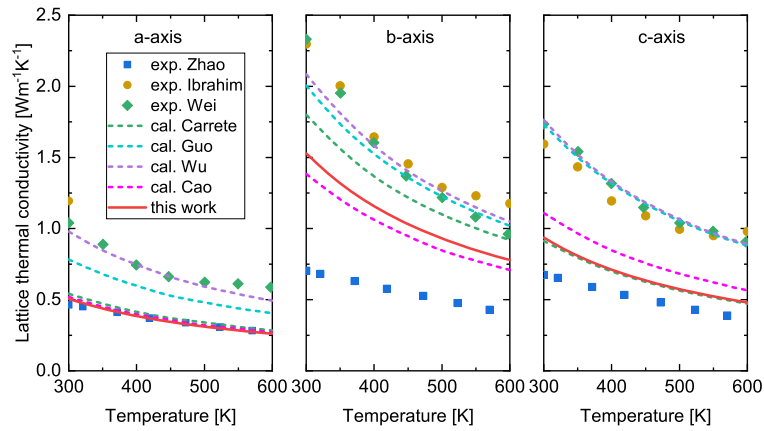
where  $n_{\mathbf{q}p}^0$  is the Bose-Einstein distribution of  $\mathbf{q}p$  mode,  $\omega_{\mathbf{q}p}$  is the phonon frequency,  $v_{\mathbf{q}p}$  is the phonon velocity,  $\mathbf{F}_{\mathbf{q}p}$  is the mean free displacement obtained from iterative solution of phonon BTE [2,31].

The DFT and DFPT calculations were implemented using Quantum ESPRESSO [32], and the norm-conserving pseudopotentials with Perdew-Burke-Ernzerhof (PBE) exchange-correlation functional were adopted. Although previous calculation reported that the experimental crystal structure gives a phonon dispersion more consistent with experimental measurements than that of relaxed structure with PBE [33], we still follow the standard first-principles calculations processes to fully relax the crystal structure. Since the spin orbital coupling has been proven negligible [33], it is not included in the calculations. For  $\{\alpha, \sigma, \kappa_e\}$ , the calculations were initially conducted on  $6 \times 10 \times 10$   $\mathbf{k}$  and  $3 \times 5 \times 5$   $\mathbf{q}$  grids to obtain the hamiltonian matrix, dynamic matrix and electron-phonon coupling matrix. The Wannier function interpolation in EPW package [8] was used to acquire the quantities on dense grids of  $24 \times 48 \times 48$  for  $\mathbf{k}$  and  $\mathbf{q}$  points in solving electron BTE. For  $\kappa_l$ , the second-order interatomic force constants (IFCs) obtained on  $3 \times 5 \times 5$   $\mathbf{q}$  grids in above DFPT calculations was used, while the third-order IFCs was calculated using  $2 \times 4 \times 4$  supercells with a cutoff distance of 0.70 nm, which is long enough to eliminate the inaccuracy of force truncation [34]. The ShengBTE [2] was used for iterative solution of phonon BTE with  $24 \times 24 \times 24$   $\mathbf{q}$  points. These above parameter settings are enough for the convergence of quantities above room temperature.

## 3. Results and discussions

In the weak doping limit where the defects-caused scattering is negligible as compared to the other types of relevant scattering, the calculations do not need to involve modeling of defects. However, in the doping range technologically relevant to thermoelectric materials, defects scattering affects the carriers (both electrons and phonons) lifetimes. From the Wiedemann-Franz law, the Lorenz number is defined by  $L = \kappa_e / (\sigma T)$ , then the  $ZT$  can be rewritten as

$$ZT = \frac{\alpha^2 n_c e T}{L n_c e T + \kappa_p / \mu}, \quad (6)$$



**Fig. 1.** Lattice thermal conductivity of SnSe along its three crystallographic axes as compared with the experimental data from Zhao et al. [18], Ibrahim et al. [39], and Wei et al. [40], and the first-principles calculations by Carrete et al. [34], Guo et al. [41], Wu et al. [42], and Cao et al. [27], respectively. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

where  $n_c$  is the electron concentration,  $\mu = \sigma/(n_c e)$  is the electron mobility. Among the thermoelectric quantities,  $\alpha$  is dominated by the band structure, and thus carriers lifetimes have little effect. That is the reason that  $\alpha$  can be accurately obtained without referring to charge carriers lifetimes [20]. In single crystal,  $\mu$  is generally more sensitive to the defects than  $\kappa_l$  in most cases. For instance, in the high and moderate  $\kappa_l$  materials of BAs [35] and Si [36,37], the room-temperature  $\mu$  at carrier concentration of  $10^{19} \text{ cm}^{-3}$  is more than one order of magnitude smaller than the intrinsic values while the  $\kappa_l$  only reduces about 50% and 20%, respectively. In low  $\kappa_l$  materials like  $\text{Mg}_2\text{Si}$  [38], when the doping increases the electron concentration from  $10^{16}$  to  $10^{18} \text{ cm}^{-3}$ , the room-temperature  $\mu$  decreases about one order of magnitude but the  $\kappa_l$  does not change significantly. On the other hand, the  $\kappa_l$  of the thermoelectric material itself is low, so the change in  $\kappa_l$  after doping will not be large, such as SnSe [18,21,23,25]. This suggests that the calculated  $\kappa_l/\mu$  is more likely to be underestimated when the defect scattering is neglected.

According to the non-degenerate parabolic band model,  $L = (k_B/e)^2(5/2 + r)$  where  $r$  is the exponent of energy dependence of scattering rate as  $\tau^{-1} \propto E^{-r}$  [6,10]. For phonon-limited scattering,  $r$  is negative semidefinite. Although the scattering mechanisms caused by defects are too complicated to be accurately analyzed, two typical scatterings are clearly known to exist, *i.e.*, ionized impurity scattering and neutral defect scattering. For ionized impurity scattering,  $r$  is positive [6,17], thus ionized impurity would increase  $L$ . Recent first-principles calculations show that neutral defect results in negative  $r$  [16]. Considering the neutral defect scattering is generally weak as compared to the ionized impurity and phonon scattering [16,17],  $L$  would be little changed or underestimated when neglecting these two kinds of defects scattering. Therefore, despite the other effects of defects cannot be known, the calculated  $ZT$  from the intrinsic phonon-limited quantities may not deviate greatly from the practical  $ZT$  values. After qualitative analysis, we illustrate this point in the case of SnSe.

The low thermal conductivity is a key ingredient for the high  $ZT$  of SnSe. Experimentally measured thermal conductivity contains  $\kappa_l$  and  $\kappa_c$ . In low charge-carrier concentration limit, the contribution of  $\kappa_c$  is negligible and thus the measured thermal conductivity can be regarded as  $\kappa_l$ . Then we can collect three sets of experimental  $\kappa_l$  for SnSe along its  $a$ ,  $b$  and  $c$  crystallographic axes from the literatures [18,39,40], as given in Fig. 1. The first reported thermal conductivities of SnSe in Ref. [18] are questioned not belonging to single crystal due to the low mass density of the samples therein, only about 88% of the theoretical density [43], which is later revealed stemming from the Sn vacancy [20,22]. Moreover, the measured thermal conductivity is smaller than its polycrystal systems [44,45]. However, the unusual high thermal conductivity of polycrystal is recently demonstrated to suffer from the tin

oxides ( $\text{SnO}_x$ ) covering the surface of SnSe, and the thermal conductivity can be reduced significantly in purified polycrystalline samples [46]. The other measurements on SnSe single crystals in Refs. [39] and [40] reported much higher thermal conductivities than those of Ref. [18].

The  $\kappa_l$  obtained by first-principles calculations in previous studies are also plotted for comparison [34,41,42]. These calculations are all implemented in VASP with projector augmented wave (PAW) pseudopotentials. Guo et al. [41] used  $2 \times 3 \times 3$  supercell with local density approximation (LDA) pseudopotentials, while Wu et al. [42] used  $2 \times 4 \times 4$  supercell with PBE pseudopotentials and the van der Waals (vdW) interaction was additionally included. Comparing to the results of Guo et al., Wu et al. presented similar  $\kappa_l$  along the  $b$  and  $c$  axes but obviously higher  $\kappa_l$  along the  $a$  axis. Carrete et al. [34] adopted  $3 \times 5 \times 5$  supercell with PBE pseudopotentials while Cao et al. [27] used  $2 \times 3 \times 3$  supercell. The obtained  $\kappa_l$  are obvious lower than those of Guo et al. and Wu et al. in all three directions. The discrepancy of calculations probably comes from the different structural and computational parameters. Herein, the  $\kappa_l$  are more consistent with those of Carrete et al. [34] and Cao et al. [27], all of them are implemented with PBE and without vdW correction. The calculated  $\kappa_l$  are 0.5, 1.5 and 0.9  $\text{Wm}^{-1}\text{K}^{-1}$  at room temperature along the  $a$ ,  $b$  and  $c$  axes, respectively.

Fig. 2 shows the calculated  $ZT$  of  $n$ -type SnSe at different temperatures. The  $ZT$  at carrier concentrations of  $10^{17}$ ,  $10^{18}$ ,  $10^{19}$ , and  $10^{20} \text{ cm}^{-3}$  as well as the maximum values obtained in calculations are plotted. The maximum  $ZT$  corresponds to a carrier concentration around  $10^{19} \text{ cm}^{-3}$  for all three axes. As temperature increases from 300 to 600 K, the calculated maximum  $ZT$  of  $n$ -type SnSe along  $a$ ,  $b$  and  $c$  axes increase monotonously from 0.48, 0.38, and 0.43 to 1.90, 0.91, and 1.21, respectively. The  $n$ -type SnSe was firstly synthesized by Duong et al. [22] with bismuth substituting Sn sites, yielding a carrier concentration of  $2.1 \times 10^{19} \text{ cm}^{-3}$ , and the measurements give nearly identical high  $ZT$  along  $b$  and  $c$  axes, while the  $a$ -axis  $ZT$  is relatively small. The experimental  $ZT$  is much smaller than the predicted maximum value at room temperature, but close to that with increasing temperatures along  $b$  and  $c$  axes. In contrast to the findings of Duong et al. [22], the first-principles calculations suggest that the  $a$  axis should have higher  $ZT$  than  $b$  and  $c$  axes. Chang et al. [19] successfully synthesized  $n$ -type SnSe through the bromine doping at a concentration of  $1.2 \times 10^{19} \text{ cm}^{-3}$ . They found much higher  $ZT$  along  $a$  axis, with a value of about 0.25 at room temperature. Subsequently, Su et al. [24] enhanced the  $a$ -axis  $ZT$  of bromine-doped SnSe by additionally introducing lead alloying, and reported a record high  $ZT$  along  $a$  axis using chlorine dopant instead of bromine. It can be seen that the highest values of measured  $ZT$  for  $n$ -type SnSe are near the maximum  $ZT$  predicted by the first-principles calculations.

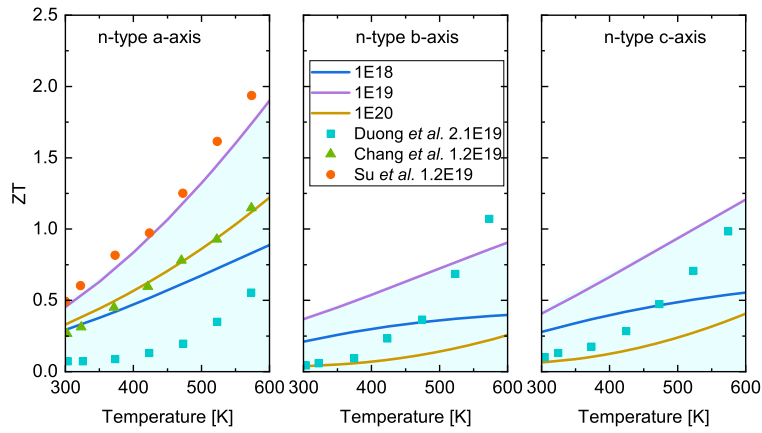


Fig. 2. Calculated  $ZT$  of  $n$ -type SnSe at different carrier concentrations and temperatures as compared with experimental data from Duong et al. [22], Chang et al. [19], and Su et al. [24]. The upper bound of light blue background indicates the maximum  $ZT$  obtained in calculations.

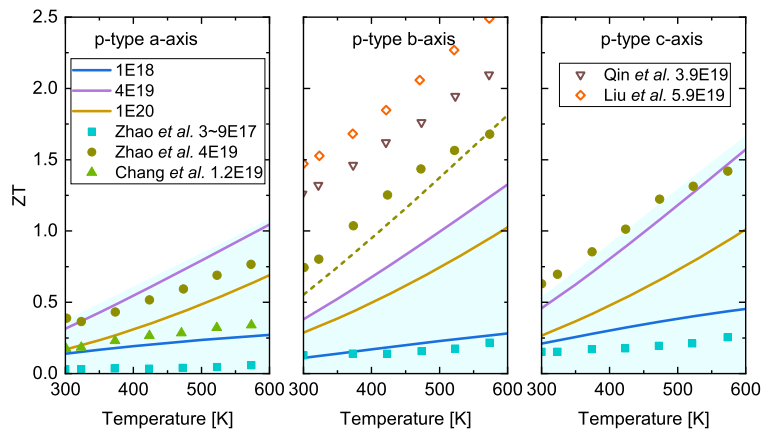


Fig. 3. Calculated  $ZT$  of  $p$ -type SnSe at different carrier concentrations and temperatures as compared with experimental data from Zhao et al. [18,21], Chang et al. [19], Qin et al. [23], and Liu et al. [25]. The upper limit of light blue background indicates the maximum  $ZT$  obtained in calculations. The dashed line gives the  $b$ -axis  $ZT$  calculated with  $k_i$  of  $c$  axis. Note that the open symbols indicate that the band structure of experimental samples has significantly changed, which is beyond the scope of this article.

The calculated  $ZT$  of  $p$ -type SnSe are compared with experimental values in Fig. 3. The maximum  $ZT$  along  $a$ ,  $b$  and  $c$  axes from calculations are 0.37, 0.39, and 0.53 at 300 K, which increase to 1.08, 1.33, and 1.67 at 600 K, respectively. The pristine SnSe reported by Zhao et al. [18] exhibits hole concentrations of only about  $3\sim 9\times 10^{17} \text{ cm}^{-3}$  in the temperature range of 300-600 K. Thus, although high  $ZT$  is found in  $Cmcm$  phase, the  $ZT$  for  $Pnma$  SnSe is smaller than 0.25 in all three axes. Soon afterwards, Zhao et al. [21] achieved heavy hole doping of  $4\times 10^{19} \text{ cm}^{-3}$  in SnSe single crystals through sodium doping, and the  $ZT$  of  $p$ -type SnSe are significantly increased. The  $ZT$  measured along  $a$  and  $c$  axes are close to the maximum values of calculations, whereas the measured  $ZT$  along  $b$  axis is much higher than the calculated upper bound. It is noticed that the measured  $\kappa_l$  of SnSe along the  $b$  axis are almost identical to that of the  $c$  axis [18,21], which is contrary to the calculated results with strong anisotropy. If the  $c$ -axis  $\kappa_l$  are used in the calculations of  $b$ -axis  $ZT$ , the obtained values can be significantly increased and close to the experimental values, as shown by the dashed line in Fig. 3. Recently, the  $ZT$  of  $p$ -type SnSe along  $b$  axis has been further improved by lead alloying [23] and copper occupying [25]. Different from the sodium doping which only generates a high hole concentration but remains the crystal lattice structure or the band structure unchanged [25,47], the lead alloying and copper occupying can effectively modulate the crystal structure and promote multi-band synergism which further optimize the electrical transport properties [23,25]. Since the absence of such strong band engineering effect in the cal-

culations, it is understandable that the experimental values are much higher than the calculated maximum.

The contour maps of  $ZT$  depending on temperature and carrier concentration for SnSe are given in Fig. 4. It shows that the  $n$ -type SnSe observes higher  $ZT$  along  $a$  axis than that of  $b$  and  $c$  axes, and the  $a$ -axis  $ZT$  goes beyond unity in a large carrier concentration and temperature range. For  $p$ -type SnSe, the higher  $ZT$  appears in the  $b$  and  $c$  directions, and the highest  $ZT$  that can be achieved is smaller than  $n$ -type SnSe due to much higher  $\kappa_l$  along  $b$  and  $c$  axes. In addition, the electron concentrations corresponding to the maximum  $ZT$  are about  $6\times 10^{18}\sim 1\times 10^{19} \text{ cm}^{-3}$ , while the optimal hole concentrations are about  $2\sim 4\times 10^{19} \text{ cm}^{-3}$  in the temperature range of 300-600 K. These concentrations are close to the measured concentrations of experimental SnSe samples. This implies that the calculations help to identify the optimal doping concentration to some extent.

After direct comparison between the phonon-limited  $ZT$  and experimental values, we examine the doping influence on the calculated  $ZT$  using semiempirical models. The  $n$ - and  $p$ -type SnSe are assumed by substituting Bi and Na at Sn sites, respectively [21,22]. The dopant interaction in electrical transport is considered as ionized impurity scattering using Brooks and Herring model [48], while its scattering to phonons is determined by mass disorder using the scheme as isotope scattering [12] (cf. S1 in the supplementary material). Fig. 5 presents the calculated  $\mu$ ,  $\kappa_l$ , and  $ZT$  at 500 K under different doping concentrations with and without impurity scattering. At a dopant concentration of  $10^{19} \text{ cm}^{-3}$ , the  $\mu$  in  $n$ - and  $p$ -type SnSe is decreased by 10-16%,

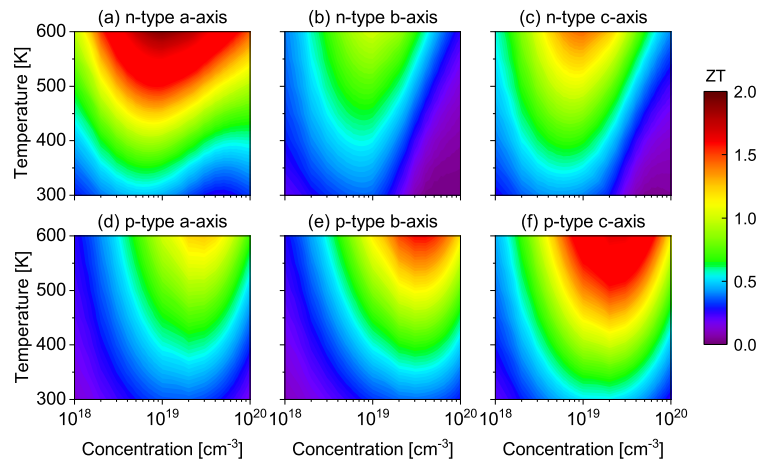


Fig. 4. Contour map of  $ZT$  as a function of temperature and carrier concentration for  $n$ - and  $p$ -type SnSe along  $a$ ,  $b$  and  $c$  axes.

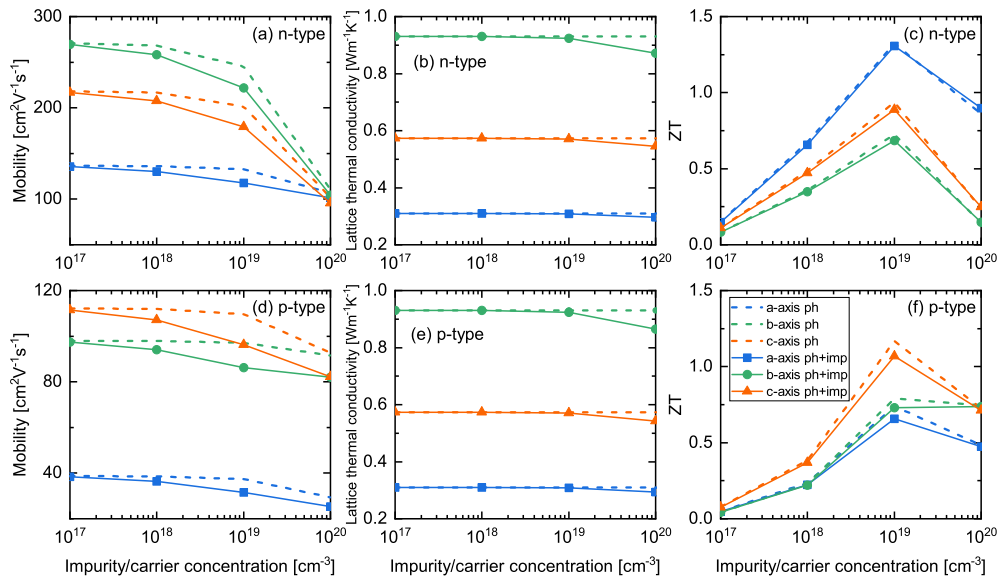


Fig. 5. Calculated (a, d) mobility, (b, e) lattice thermal conductivity, and (c, f)  $ZT$  for  $n$ - and  $p$ -type SnSe at 500 K under different doping concentrations with and without impurity scattering.

while the decrease of  $\kappa_l$  is within 1%. The reduction by  $10^{20} \text{ cm}^{-3}$  doping is 5-14% for  $\mu$  and 5-7% for  $\kappa_l$ . This is in line with the general experience stated above that  $\mu$  is more sensitive to defects than  $\kappa_l$ . Correspondingly, the decrease of  $ZT$  is smaller than 10% and 6% in all three directions under  $10^{19}$  and  $10^{20} \text{ cm}^{-3}$  doping concentrations, respectively. Although the doping scattering is only approximately estimated, the results still indicate to a certain extent that the change of calculated  $ZT$  by doping defects is somewhat weakened compared to the electrical transport properties. Therefore, the speculation about  $ZT$  in Eq. (6) can be confirmed to a considerable extent.

Adjusting the pressures on the crystals has been long recognized as an important scheme to modulate their lattice structure and thus electrical and thermal transport properties. By applying pressures on moderately hole-doped SnSe with carrier concentration of about  $2 \times 10^{18} \text{ cm}^{-3}$ , Nishimura et al. [28] experimentally found that the in-plane ( $b$ - $c$  plane) thermoelectric power factor,  $PF = \alpha^2 \sigma T$ , obtained a large enhancement over a wide temperature range from 10 to 300 K. The room-temperature  $PF$  increases from  $5 \mu\text{Wcm}^{-1}\text{K}^{-2}$  at ambient pressure to  $12 \mu\text{Wcm}^{-1}\text{K}^{-2}$  at 1.6 GPa pressure. Here, we calculated the thermoelectric properties of  $n$ - and  $p$ -type SnSe at pressures of 1.6 GPa and 3.2 GPa. As expected, the lattice constants decrease with increasing

pressures, from basic parameters of  $a = 11.72 \text{ \AA}$ ,  $b = 4.21 \text{ \AA}$ ,  $c = 4.49 \text{ \AA}$  to  $a = 11.51 \text{ \AA}$ ,  $b = 4.18 \text{ \AA}$ ,  $c = 4.37 \text{ \AA}$  under 1.6 GPa pressure and  $a = 11.37 \text{ \AA}$ ,  $b = 4.16 \text{ \AA}$ ,  $c = 4.28 \text{ \AA}$  under 3.2 GPa pressure.

Fig. 6 shows the pressure dependent  $PF$  of  $n$ - and  $p$ -type SnSe as a function of carrier concentrations at room temperature. For  $p$ -type SnSe, with increase of pressure, the  $PF$  along the  $b$  axis increases significantly, followed by the  $c$  axis and  $a$  axis but much less than the increase in the  $b$  axis. At hole concentration of  $2 \times 10^{18} \text{ cm}^{-3}$ , the calculated room-temperature  $PF$  along  $b$  axis is  $8.5 \mu\text{Wcm}^{-1}\text{K}^{-2}$  with no pressure applied and  $13.4 \mu\text{Wcm}^{-1}\text{K}^{-2}$  under 1.6 GPa, which are roughly consistent with the experiment findings [28]. The maximum  $PF$  of  $p$ -type SnSe along  $b$  axis is enhanced from 30 to 47 and  $90 \mu\text{Wcm}^{-1}\text{K}^{-2}$  by 1.6 and 3.2 GPa pressures, respectively. In contrast, for  $n$ -type SnSe, the pressure-induced  $PF$  enhancement is mainly manifested in the  $a$ -axis direction, but there is no significant improvement in the  $b$  and  $c$  axes. At an electron concentration of  $10^{19} \text{ cm}^{-3}$ , the room-temperature  $PF$  can be increased from 12 to  $34 \mu\text{Wcm}^{-1}\text{K}^{-2}$  by 1.6 GPa pressure and to  $59 \mu\text{Wcm}^{-1}\text{K}^{-2}$  by 3.2 GPa pressure. The anisotropic enhancement of  $PF$  can be understood from the nonmonotonic influence of pressure on  $\sigma$  and  $\alpha$ . For  $p$ -type SnSe, as pressure increases, the  $\sigma$  in all three axes increase, but the  $\alpha$  in  $a$  and  $c$  axes decrease obviously (Fig. S1 and S2 in

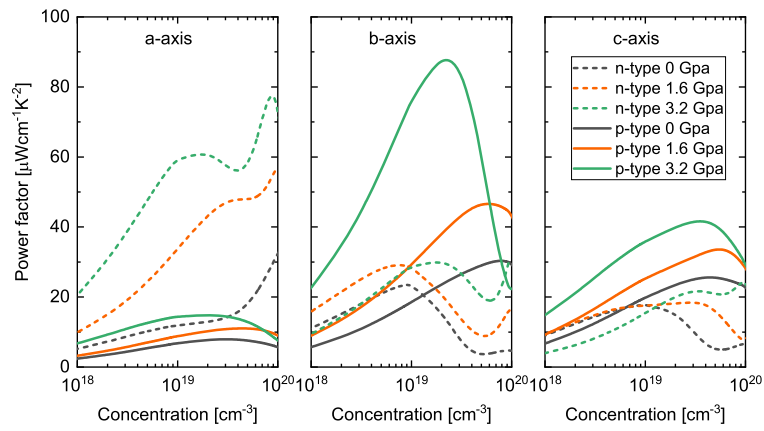


Fig. 6. Calculated room-temperature  $PF$  of  $n$ - and  $p$ -type SnSe at different carrier concentrations and pressures.

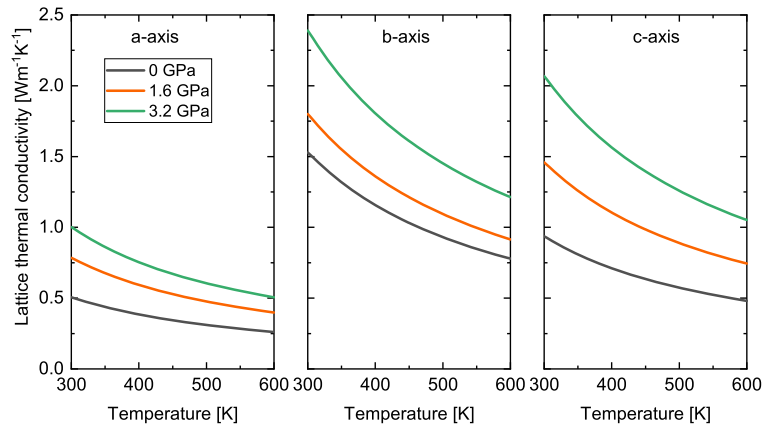


Fig. 7. Calculated  $\kappa_l$  of SnSe at different temperatures and pressures.

the supplementary material). As a result, only the  $b$ -axis  $PF$  has significant enhancement while the increase in other axes is weak. The change of  $\sigma$  and  $\alpha$  in  $n$ -type SnSe is different from that of  $p$ -type SnSe. When a pressure of 3.2 GPa is applied, the  $\alpha$  increases in all axes, while the  $\sigma$  only increases in  $a$  axis but reduces in  $b$  and  $c$  axes (Fig. S1 and S2 in the supplementary material). Then, the  $PF$  of  $n$ -type SnSe along  $a$ -axis direction gains a large enhancement but the other axes do not. In addition, following the change trend of  $\sigma$  under increasing pressure, the  $\mu$  at low concentration of  $p$ -type SnSe is increased in all three axes. In  $n$ -type SnSe, the  $\mu$  increases in  $a$  axis but decreases in  $c$  axes, while the  $b$ -axis  $\mu$  is increased by 1.6 GPa but decreased by 3.2 GPa pressure (Fig. S4 in the supplementary material).

In addition to the electrical transport properties, the pressure also alters the phonon transport of SnSe. Fig. 7 presents the calculated  $\kappa_l$  of SnSe at different pressures. It can be seen that the  $\kappa_l$  increases monotonically in all three axes as pressure increases up to 3.2 GPa, in consistent with the results of Ref. [27] for SnSe and its analogue of SnS [49]. At 1.6 GPa pressure, the room-temperature  $\kappa_l$  are 0.8, 1.8 and 1.5  $\text{Wm}^{-1}\text{K}^{-1}$  along  $a$ ,  $b$  and  $c$  axes, respectively, while the values are 1.1, 2.4 and 2.1  $\text{Wm}^{-1}\text{K}^{-1}$  under 3.2 GPa pressure. The relation of  $\kappa_l^b > \kappa_l^c > \kappa_l^a$  remains when the pressures applied. But the anisotropy of  $\kappa_l^b/\kappa_l^c$  decreases from 1.7 with no pressure applied to 1.2 and 1.1 under 1.6 and 3.2 GPa, respectively. Following the usual rules, the pressure compresses the lattice structure, which stiffens the phonon dispersion and thus increases the phonon velocity. This can be seen from the averaged phonon velocities of SnSe under different pressures (Fig. S5 in the supplementary material). The increased pressure also decreases the anharmonicity of phonons below 5  $\text{rad}\cdot\text{ps}^{-1}$  and slightly increases the anharmonicity above this frequency, as revealed by the Grüneisen

parameters (Fig. S6(a) in the supplementary material). The scattering phase space for phonons below 5  $\text{rad}\cdot\text{ps}^{-1}$  is suppressed by pressures, and it is increased for phonons between 5 and 12  $\text{rad}\cdot\text{ps}^{-1}$ , after which it again decreases for phonons with frequency up to 20  $\text{rad}\cdot\text{ps}^{-1}$  (Fig. S6(b) in the supplementary material). Thus, the scattering rates, proportional to the anharmonicity and scattering phase space, of phonons below 20  $\text{rad}\cdot\text{ps}^{-1}$  are overall decreased slightly by pressures (Fig. S6(c) in the supplementary material). Meanwhile, these phonons with frequency smaller than 20  $\text{rad}\cdot\text{ps}^{-1}$  contribute most of the  $\kappa_l$ , more than 62%, 70% and 73% under pressures of 0 GPa, 1.6 GPa and 3.2 GPa, respectively (Fig. S7 in the supplemental material). As a result, the increased phonon velocity and the reduced phonon scattering rate raise the  $\kappa_l$  of SnSe. Therefore, although the pressure enormously improves the  $PF$  of  $n$ - and  $p$ -type SnSe along  $a$  and  $b$  axes, respectively, the increased  $\kappa_l$  gives rise to a negative influence on the  $ZT$  value.

Fig. 8 shows the influences of pressures on the  $ZT$  of  $n$ - and  $p$ -type SnSe at room temperature. For  $n$ -type SnSe, the pressure induced enhancement of  $PF$  in the  $b$  and  $c$  axes is small. Although the  $b$ - and  $c$ -axis  $\kappa_e$  are decreased by pressure, at  $10^{19} \text{ cm}^{-3}$  concentration, the  $\kappa_e$  is smaller than 0.5  $\text{Wm}^{-1}\text{K}^{-1}$  (Fig. S3 in the supplementary material), which means that the total thermal conductivity is dominated by  $\kappa_l$ . Therefore, the small enhancement of  $PF$  is not enough to offset the deterioration from the increased  $\kappa_l$ , so the  $ZT$  value decreases slightly with the increase of pressure. Nevertheless, in the  $a$ -axis direction, the huge enhancement in  $PF$  still guarantees an increase in the  $ZT$  value. The maximum  $ZT$  of  $n$ -type SnSe along  $a$  axis is improved to 0.87 at an electron concentration around  $10^{19} \text{ cm}^{-3}$  by 1.6 GPa pressure, while the value at 3.2 GPa pressure is 1.2 around the concentration of  $8 \times 10^{18} \text{ cm}^{-3}$ . The effect of pressure on  $p$ -type SnSe shows similar changes due

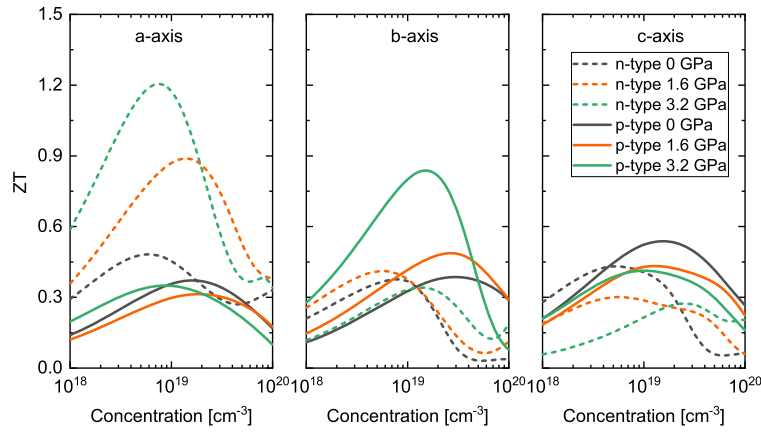


Fig. 8. Calculated room-temperature  $ZT$  of  $n$ - and  $p$ -type SnSe at different carrier concentrations and pressures.

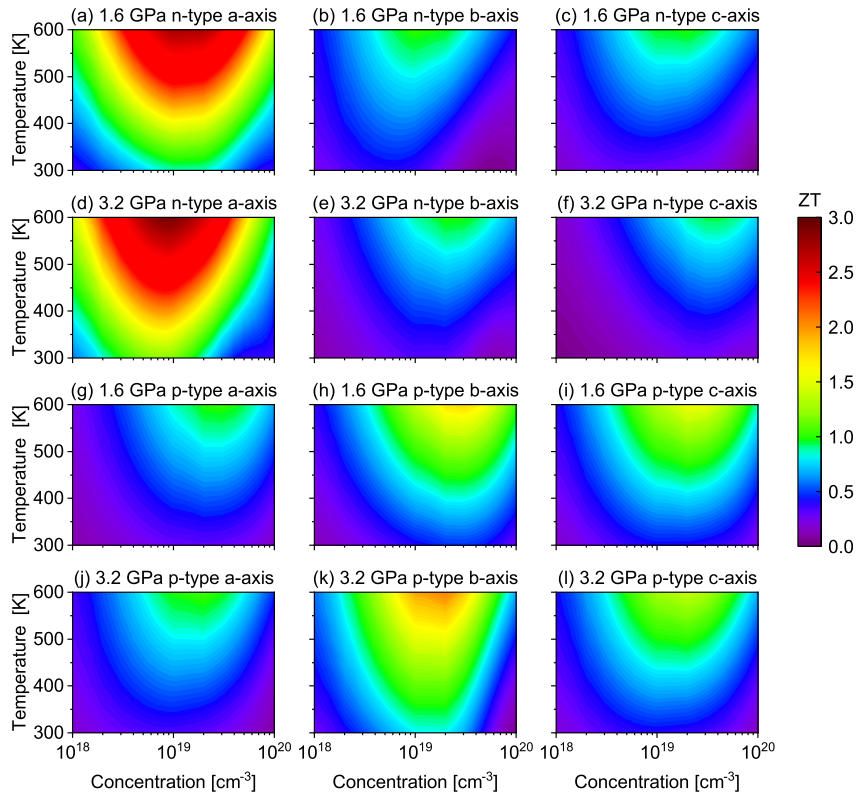


Fig. 9. Contour map of  $ZT$  as a function of temperature and carrier concentration for SnSe under 1.6 and 3.2 GPa pressures. (a-f) are for  $n$ -type SnSe and (g-l) are for  $p$ -type SnSe along  $a$ ,  $b$  and  $c$  axes.

to increased  $\kappa_e$  and  $\kappa_l$ , but mainly has significant improvement in the  $b$ -axis  $ZT$ . The maximum  $ZT$  of  $p$ -type SnSe along  $b$  axis are 0.49 and 0.82 under pressures of 1.6 and 3.2 GPa, respectively. The corresponding hole concentrations are  $3 \times 10^{19}$  and  $2 \times 10^{19} \text{ cm}^{-3}$ , respectively. The anisotropy of  $ZT$  under pressures becomes stronger as compared to that of original SnSe [27]. Applying 1.6 and 3.2 GPa pressures on  $n$ -type SnSe can enhance the  $ZT$  along  $a$  axis by 81% and 150%, respectively. Likewise, the increases of  $ZT$  for  $p$ -type SnSe along  $b$  axis are 26% and 110%, respectively.

The temperature and carrier concentration dependence of  $ZT$  for  $n$ - and  $p$ -type SnSe under pressures of 1.6 and 3.2 GPa are depicted in Fig. 9. It can be seen that the enhancement of  $ZT$  by increased pressure persists from low to high temperatures, mainly on the  $a$  axis of  $n$ -type SnSe and  $b$  axis of  $p$ -type SnSe. At 600 K, the  $a$ -axis  $ZT$  for  $n$ -type SnSe are 2.7 and 2.9 under pressures of 1.6 and 3.2 GPa, respectively, while

the  $ZT$  are 1.6 and 1.8 for  $p$ -type SnSe along  $b$  axis. By applying pressure up to 3.2 GPa, the  $ZT$  at 600 K are increased by 53% and 38% for  $n$ -type SnSe along  $a$  axis and  $p$ -type SnSe along  $b$  axis, respectively. For  $p$ -type SnSe, the pressure induces the  $ZT$  along  $b$  axis to become larger than that of  $c$  axis, due to the reduced anisotropy of  $\kappa_l$  of  $b$  and  $c$  axes. In addition, the optimal carrier concentration corresponding to the highest  $ZT$  does not change significantly with the change of temperature and pressure. The calculated optimal concentrations under 1.6 and 3.2 GPa for  $n$ -type  $a$ -axis  $ZT$  are from  $6 \times 10^{18}$  to  $1 \times 10^{19} \text{ cm}^{-3}$  in the temperature range of 300-600 K, while the optimal concentrations are from  $2 \times 10^{19}$  to  $4 \times 10^{19} \text{ cm}^{-3}$  for  $p$ -type  $b$ -axis  $ZT$ . These calculated concentrations are close to the doping concentrations that have been experimentally achieved so far, indicating that it is feasible to further induce the enhancement of  $ZT$  for SnSe by pressure.

#### 4. Conclusions

The intrinsic electrical and thermal transport properties of *Pnma* SnSe were obtained from the iterative Boltzmann transport equations combined with first-principles calculated electron-phonon and phonon-phonon limited scatterings. The qualitative analysis argues that the *ZT* without including the defect scattering may not deviate greatly from the practical values. The phonon-limited *ZT* of *n*- and *p*-type SnSe were calculated, which indeed a certain degree of quantitatively reproduced the experimental values. The reasonable agreement between intrinsic calculations and practical measurements verifies that the phonon-limited *ZT* can serve as a good assessment of actual thermoelectric performance of single crystals. This provides an effective way to predict the *ZT* promise of thermoelectric materials, and also suggests optimal carrier concentration ranges for experimental synthesization. Then, the influence of pressure on the thermoelectric performance of SnSe was investigated. The lattice thermal conductivity of SnSe increases monotonically as pressure increases due to increased phonon velocity and decreased phonon scattering. The thermoelectric power factor of SnSe is also increased by pressure, and is significant in the *a* axis of *n*-type SnSe and *b* axis of *p*-type SnSe, but weak along other axial directions. The huge enhancement in thermoelectric power factor offsets the deterioration from the increased lattice thermal conductivity, yielding an increase in the *ZT*. At room temperature, the *a*-axis *ZT* of *n*-type SnSe is increased from 0.48 to 0.87 and 1.20, while the *b*-axis *ZT* of *p*-type SnSe is increased from 0.39 to 0.49 and 0.82, by applying pressures of 1.6 and 3.2 GPa, respectively. The pressure induced enhancement of *ZT* hold at high temperatures. The corresponding electron and hole carrier concentrations for the optimal thermoelectric performance are around  $10^{19} \text{ cm}^{-3}$  in the temperature range of 300-600 K and do not change significantly by pressures.

#### CRedit authorship contribution statement

**Jinlong Ma:** Conceptualization, Investigation, Software, Writing – original draft. **Yani Chen:** Writing – review & editing. **Wu Li:** Conceptualization, Validation, Writing – review & editing. **Xiaobing Luo:** Resources, Supervision.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### Data availability

Data will be made available on request.

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#### Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.ijheatmasstransfer.2023.125063>.

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