Modeling the Light Extraction Efficiency of Bi-Layer Phosphors in White LEDs

Run Hu, Bin Cao, Yong Zou, Yongming Zhu, Sheng Liu, *Senior Member, IEEE*,
and Xiaobing Luo, *Senior Member, IEEE*

***Abstract*— In this letter, we modeled the bi-layer phosphors in light-emitting diode packages and optimized the concentration gradient to enhance the light extraction efficiency (LEE). The modeling processes were presented in detail. Based on our bi-layer model, we changed the concentration from 0.05 to 0.4 g/cm^3 in both the upper and lower phosphor layers. It was found that when the concentration gradient between the upper layer and lower layer increases from 0.05:0.4 to 0.4:0.05, the LEE would be enhanced as much as 17.63%. The trends were validated by experiments. Some potential ways were proposed to realize this kind of phosphor configuration where the concentration of the upper layer was larger than that of the lower layer.**

***Index Terms*—Light emitting diode (LED), phosphor, light extraction efficiency.**

I. INTRODUCTION

DUE to the extraordinary features, blue gallium-nitride (GaN) light-emitting diodes (LEDs) are being used as the dominant excitation source to pump the down-converting phosphors to generate white light [1]–[6]. White LEDs are widely used in decorative lighting, backlighting, vehicle front lighting, street lighting, and even general lighting. Among the current state-of-the-art approaches to generate white light based on blue LED chips, phosphor-converting scheme is the most widely adopted one. Plainly speaking, the phosphor-converting scheme is that the phosphor particles absorb part of the blue light from the LED chip, then down-convert part of the absorbed blue light into yellow emission. The mixing of the transmitted blue light and yellow light generates white light by color mixing [7]. In the phosphor-converting processes, light absorption, light scattering and light conversion happen simultaneously.

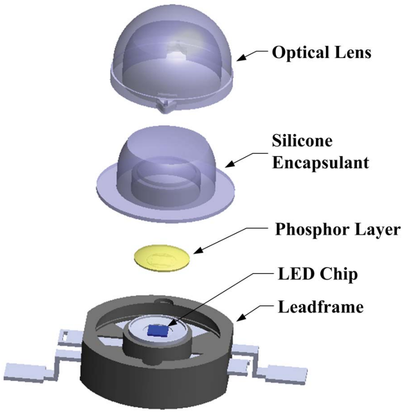

Fig. 1. Typical global configuration of LED package.

Nonetheless, further breakthroughs in light extraction efficiency (LEE) are still urgent due to the increasing demand of the market and applications [8], [9]. The enhancement of LEE not only can strengthen the luminous efficacy, but also benefit the thermal management of LED packages. A low LEE means more rays are trapped and absorbed and less rays can be extracted from the package. When more light is trapped inside the package or back-reflected to the LED chips, the junction temperature of the chip increases, which may degrade the light output and deteriorate the reliability and lifetime of LED packages [10], [11]. Therefore, how to pump out the light and enhance the LEE is the key parameter in designing the structures of LED packages.

According to the typical global configuration of LED package, as shown in Fig. 1, the total LEE of the LED package is the product of the LEEs in chip, phosphor layer, silicone encapsulant, and optical lens. The LEE in phosphor layer may be the most important one because the light conversion process happens there. It is reported that phosphor thickness, concentration, particle size, geometry, amount, and arrangement, play important roles in enhancing the LEE and reducing the light trapping in LEDs [12]–[18]. To enhance the LEE in phosphor, some novel structures or methods have been proposed, such as the scattered photon extraction (SPE) package [19], remote phosphor with hemispherical dome [16], [20], large micro-size cube phosphors [21]. Besides, multi-layer phosphor configurations are also reported as potential ways to enhance the LEE. You *et al.* combined the separate red and yellow phosphor layers to enhance the LEE by experiments [22]. Li *et al.* proposed a double-layer model to analyze

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R. Hu and X. Luo are with the School of Energy and Power Engineering and Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: hurun@hust.edu.cn; luoxb@mail.hust.edu.cn).

B. Cao is with School of Optic and Electrical Information, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: caobin36@gmail.com).

Y. Zou and Y. Zhu are with the School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: 823366619@qq.com; zymapj@163.com).

S. Liu is with School of Mechanical Science and Engineering and Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: victor_liu63@126.com).

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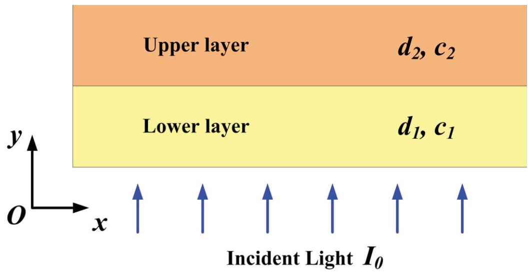

Fig. 2. Light propagation on bi-layer phosphors. c_1 and c_2 are the phosphor concentration, and d_1 and d_2 are the thickness of each phosphor layer, respectively. The intensity of the incident blue light is I_0.

the LEE, but the light scattering processes were neglected and simplified in their model [23]. Due to the complication of the light absorption, scattering and conversion processes in the phosphors, most studies about the phosphor configuration optimizations are based on the experiments [19], [21] or based on the ray-tracing simulations using the commercial software packages [3], [5], [6], [12], [13].

In this letter, we developed a model to analyze the LEE of bi-layer phosphors by modifying the Kubelka-Munk theory. The detailed modeling processes were presented. Based on the bi-layer model, we optimized the concentration gradient in the bi-layer phosphors to enhance the LEE. Experiments were conducted for validation as well. Some potential methods were proposed to realize the optimized phosphor configurations.

II. MODELING METHODOLOGY

When a blue light beam projects on the bi-layer phosphor configuration with different thicknesses and concentrations, as shown in Fig. 2, light absorption, scattering, and conversion take place among the phosphor particles in both two layers. Since the two layers will interact on each other, for instance, both blue light or yellow light would be scattered between them, we can simplify the interactions by separating the two layers apart and analyze each layer respectively, as shown in Fig. 3. In Fig. 3, the functions I and J denote the forward scattering light intensity and backscattering light intensity, respectively. The subscript B and Y denote the blue light and yellow light, respectively. The subscript numbers 0–3 denote the boundaries of the two layers. For each layer, we assumed that the phosphor layer is continuous optical medium, thus the Kubelka-Munk theory can be applied when the optical properties are just determined by the absorption and scattering coefficients. It is seen that in Kubelka-Munk theory, the light conversion processes are neglected.

In our previous study, we modified the Kubelka-Munk theory and proposed a single-layer model by taking account of light absorption, scattering and conversion in phosphors simultaneously [24]. When separating the two layers apart, we could apply the proposed single-layer model in each layer, respectively. For the lower layer, as shown in Fig. 3(b), we could obtain the forward scattering and backscattering light intensities at the boundaries as follows.

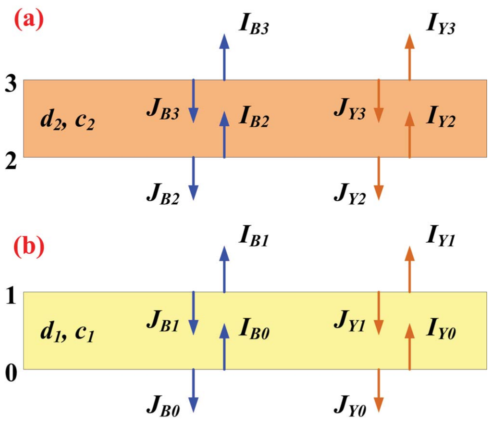

Fig. 3. Schematic of forward scattering and backscattering in the two layers separately: (a) upper layer and (b) lower layer.

For blue light at the boundaries

$$\begin{cases} I_{B0} = A_L(1-\beta_L) + B_L(1+\beta_L) \\ J_{B0} = A_L(1+\beta_L) + B_L(1-\beta_L) \\ I_{B1} = A_L(1-\beta_L)e^{\alpha_L d_L} + B_L(1+\beta_L)e^{-\alpha_L d_L} \\ J_{B1} = A_L(1+\beta_L)e^{\alpha_L d_L} + B_L(1-\beta_L)e^{-\alpha_L d_L}. \end{cases} \tag{1}$$

For yellow light at the boundaries

$$\begin{cases} I_{Y0} = C_L(1-\nu_L) + D_L(1+\nu_L) \\ \qquad + \frac{2\eta_L a_{BL}}{\nu_L(\mu_L^2-\alpha_L^2)}[A_L(\mu_L - \nu_L\alpha_L) + B_L(\mu_L + \nu_L\alpha_L)] \\ J_{Y0} = C_L(1+\nu_L) + D_L(1-\nu_L) \\ \qquad + \frac{2\eta_L a_{BL}}{\nu_L(\mu_L^2-\alpha_L^2)}[A_L(\mu_L + \nu_L\alpha_L) + B_L(\mu_L - \nu_L\alpha_L)] \\ I_{Y1} = C_L(1-\nu_L)e^{\mu_L d_L} + D_L(1+\nu_L)e^{-\mu_L d_L} \\ \qquad + \frac{2\eta_L a_{BL}}{\nu_L(\mu_L^2-\alpha_L^2)}[A_L(\mu_L - \nu_L\alpha_L)e^{\alpha_L d_L} \\ \qquad\qquad + B_L(\mu_L + \nu_L\alpha_L)e^{-\alpha_L d_L}] \\ J_{Y1} = C_L(1+\nu_L)e^{\mu_L d_L} + D_L(1-\nu_L)e^{-\mu_L d_L} \\ \qquad + \frac{2\eta_L a_{BL}}{\nu_L(\mu_L^2-\alpha_L^2)}[A_L(\mu_L + \nu_L\alpha_L)e^{\alpha_L d_L} \\ \qquad\qquad + B_L(\mu_L - \nu_L\alpha_L)e^{-\alpha_L d_L}] \end{cases} \tag{2}$$

with

$$\begin{aligned} \alpha_L &= 2\sqrt{a_{BL}(a_{BL}+2s_{BL})}, \quad \beta_L = \sqrt{a_{BL}/(a_{BL}+2s_{BL})} \\ \mu_L &= 2\sqrt{a_{YL}(a_{YL}+2s_{YL})}, \quad \nu_L = \sqrt{a_{YL}/(a_{YL}+2s_{YL})} \end{aligned} \tag{3}$$

where A, B, C, and D are the undetermined constant coefficients, which can be solved with proper boundary conditions for blue light and yellow light. η is the energy conversion coefficient from absorbed blue light to emitted yellow light. The subscript L denotes the lower layer. The subscript B and Y denote the blue light and yellow light, respectively. The subscript number 0 and 1 denote the two boundaries of the lower layer. a and s denote the absorption and scattering coefficients. The appearance of coefficient 2 in above equations is due to that the length of light path is doubled when the light scattering takes place in all the directions. Similarly, we can also obtain the forward scattering and backscattering light intensities at the boundaries of the upper layer, namely I_{B2}, J_{B2}, I_{B3}, J_{B3}, I_{Y2}, J_{Y2}, I_{Y3}, and J_{Y3}. These light intensities are general solutions and the specific solutions can be obtained with boundary conditions.

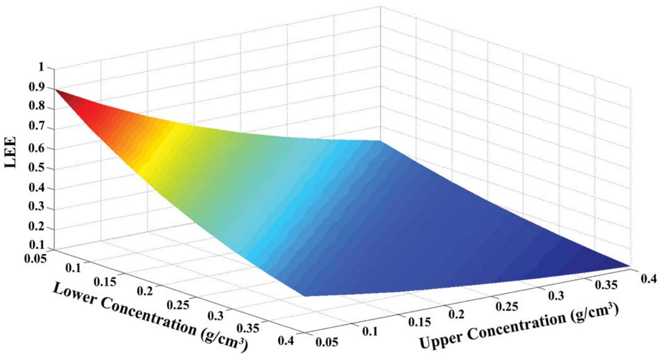

Fig. 4. LEE distribution with changing phosphor concentration gradients in the upper and lower layers.

The interactions between these two layers can be illustrated as the relationships between the light intensities on the boundaries 1 and 2 as following equations

$$\begin{cases} I_{B1} = I_{B2}, \quad J_{B1} = J_{B2} \\ I_{Y1} = I_{Y2}, \quad J_{Y1} = J_{Y2}. \end{cases} \tag{4}$$

Since the phosphors are coated on LED chips and the substrate, the backscattered blue and yellow light projected at the surfaces of chips or substrate will be reflected back to the phosphor layer. Therefore, the boundary conditions on the boundaries 0 and 3 for the blue light and yellow light are

$$I_{B0} = I_0 + \gamma_B J_{B0}, \quad J_{B3} = 0 \tag{5}$$

$$I_{Y0} = \gamma_Y J_{Y0}, \quad J_{Y3} = 0 \tag{6}$$

where γ_B and γ_Y are the reflection coefficients on the surfaces of chips or substrates. It is seen that the forward scattering and backscattering intensities for both blue light and yellow light cannot be totally solved before the awareness of the absorption and scattering coefficients of the phosphor particles. These coefficients are changed with phosphor concentrations and difficult to measure actually. In this letter, we adopted the Mie-Lorenz theory to calculate the necessary coefficients and detailed calculation processes are not addressed here [25], [26]. After obtaining the coefficients and with above equations, we can obtain all the light intensities on the boundaries for blue light and yellow light. The transmitted light includes the blue component I_{B3} and yellow component I_{Y3}, thus the LEE can be calculated as

$$LEE = (I_{B3} + I_{Y3})/I_0. \tag{7}$$

III. RESULTS AND DISCUSSION

After achieving the bi-layer phosphor model, we assigned the two layers with different concentration gradients. In our calculations, the phosphor concentration was changed from 0.05 to 0.4 g/cm^3 in both the upper and lower layers. The thickness of each layer was kept as 100 μm. The energy conversion coefficient was assumed as 0.7, which means when phosphor particle absorbs one blue photon, 0.7 yellow photons would be converted and emitted. As aforesaid, the necessary coefficients, especially the light absorption and scattering coefficients, were obtained from the Mie-Lorenz calculations. For each concentration gradient, the corresponding LEE was calculated and its distribution with exchanging the phosphor concentration gradients in the upper and lower layers was illustrated in Fig. 4. It is seen that for both the upper and lower layers, when the concentration increases, the LEE decreases monotonously. It seems that lower phosphor amount leads to higher LEE.

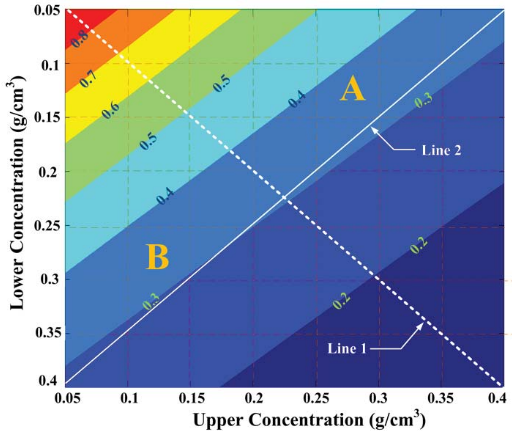

Fig. 5. Contour of LEE distribution.

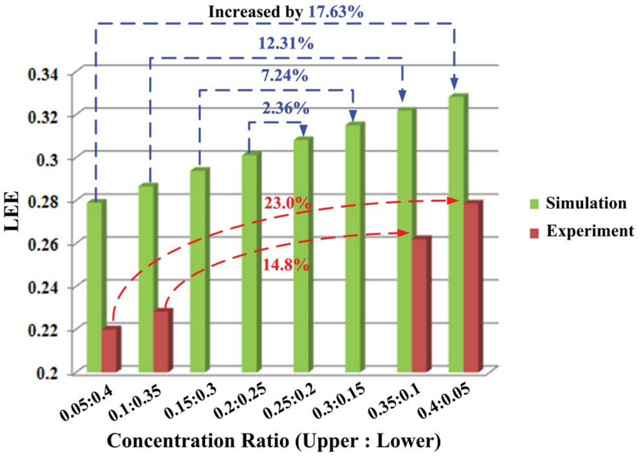

Fig. 6. LEE variations with exchanging concentration ratio between the upper layer and lower layer.

To analyze the LEE distribution further, we plotted the corresponding contour of Fig. 4 into Fig. 5. In Fig. 5, the color gradient corresponds to different values of LEE. It is also seen that the LEE decreases when the concentration increases. When drawing the dash diagonal (see Line 1), we can divide the contour into two regions, i.e. region A and region B. In region A, the concentration of upper layer is larger than that of the lower layer; whereas in region B, the situation is just on the contrary. When comparing the contour lines in region A and region B, we may find that region A and region B are not symmetrical about the dash diagonal. The LEEs of the nodes in region A are larger than those of the corresponding nodes in region B. Let's take the solid diagonal (see Line 2) as an example (see Fig. 6). Fig. 6 shows the LEE variations of the corresponding nodes on Line 2 with exchanging concentration ratio between the upper and lower layers. It is seen that when the concentration ratio increases, the LEE increases greatly. For the corresponding cases where the concentrations are exchanged between the upper layer and lower layer, the LEEs increase by 2.36%, 7.24%, 12.31%, and 17.63%, respectively when the upper concentration is larger. It is predicted from simulations that when the concentration gradient between the upper layer and lower layer increases further, the enhancement of LEE would be much more considerable. To validate our predictions, we also conducted experiments. Two pairs of white LEDs were packaged. Four kinds of concentrations were prepared and categorized into two pairs, i.e. 0.05 and 0.4 g/cm^3 in one pair, 0.1 and 0.35 g/cm^3 in the other pair. Experimental results were also shown in Fig. 6. From Fig. 6, the LEE would be enhanced by 23.0% and 14.8% respectively for the two pairs when the upper concentration was larger.

The reasons for these phenomena are: (1) the corresponding nodes in region A and region B have the same amount of phosphors. For instance, the phosphor amount of the nodes (0.05:0.4) and (0.4:0.05) is the same. (2) when the phosphor concentration of the lower layer is larger, more light would be scattered and reflected back to the chips or substrates. Thus the light energy loss increases and the transmitted light energy decreases. Therefore, when the concentration of the upper layer is larger, the LEE is larger.

Inspired by the trends in Fig. 6, when the phosphor amount is the same, we may realize the bi-layer phosphor configuration with concentration gradient to enhance the LEE, but the concentration of the upper layer should be larger than that of the lower layer. Otherwise, the LEE would be deteriorated. This phosphor configuration may be realized by dispersing two kinds of phosphors with different concentrations, or by phosphor sedimentation. If we choose to realize the configuration by phosphor sedimentation, we should put the phosphor layer upside down after the sedimentation process.

IV. CONCLUSION

In this letter, we developed a bi-layer phosphor model and optimized the concentration gradient to enhance the light extraction efficiency (LEE). In our model, we modified the Kubelka-Munk theory and took account of the light absorption, scattering, and conversion processes simultaneously. Based on our model, we changed the concentration from 0.05 to 0.4 g/cm^3 in both the upper and lower phosphor layers, and found that when the concentration gradients between the upper layer and lower layer increases from 0.05:0.4 to 0.4:0.05, the LEE would be enhanced as much as 17.63%. The trends were verified by experiments as well. It is concluded that we can enhance the LEE by realizing the phosphor configuration, in which the concentration of the upper layer is larger than that of the lower layer. This kind configuration may be realized by dispersing two kinds of phosphors or by phosphor sedimentation.

REFERENCES

[1] M. H. Crawford, “LEDs for solid-state lighting: Performance challenges and recent advances,” *IEEE J. Sel. Topics Quantum Electron.*, vol. 15, no. 4, pp. 1028–1040, Jul./Aug. 2009.

[2] R. Hu, X. B. Luo, H. Feng, and S. Liu, “Effect of phosphor settling on the optical performance of phosphor-converted white light-emitting diodes,” *J. Lumin.*, vol. 132, no. 5, pp. 1252–1256, 2012.

[3] C. Sommer, *et al.*, “The effect of the phosphor particle sizes on the angular homogeneity of phosphor-converted high-power white LED light sources,” *IEEE J. Sel. Topics Quantum Electron.*, vol. 15, no. 4, pp. 1181–1188, Jul./Aug. 2009.

[4] H. Zheng, *et al.*, “Conformal phosphor coating using capillary microchannel for controlling color deviation of phosphor-converted white light-emitting diodes,” *Opt. Express*, vol. 20, no. 5, pp. 5092–5098, 2012.

[5] R. Hu, X. B. Luo, and S. Liu, “Study on the optical properties of conformal coating light-emitting diode by Monte Carlo simulation,” *IEEE Photon. Technol. Lett.*, vol. 23, no. 22, pp. 1673–1675, Nov. 15, 2011.

[6] R. Hu, *et al.*, “Design of a novel freeform lens for LED uniform illumination and conformal phosphor coating,” *Opt. Express*, vol. 20, no. 13, pp. 13727–13737, 2012.

[7] R. Hu, *et al.*, “Near-/mid-field effect of color mixing for single phosphor-converted light-emitting diode package,” *IEEE Photon. Technol. Lett.*, vol. 25, no. 3, pp. 246–249, Feb. 1, 2013.

[8] A. Keppens, W. R. Ryckaert, G. Deconinck, and P. Hanselaer, “Modeling high power light-emitting diode spectra and their variation with junction temperature,” *J. Appl. Phys.*, vol. 108, no. 4, pp. 043104-1–043104-7, 2010.

[9] A. I. Zhmakin, “Enhancement of light extraction from light emitting diodes,” *Phys. Rep.*, vol. 498, nos. 4–5, pp. 189–241, 2011.

[10] Q. Chen, X. B. Luo, S. J. Zhou, and S. Liu, “Dynamic junction temperature measurement for high power light emitting diodes,” *Rev. Sci. Instrum.*, vol. 82, no. 8, pp. 084904-1–084904-7, 2011.

[11] R. Hu, X. B. Luo, and H. Zheng, “Hotspot location shift in the high-power phosphor-converted white light-emitting diode packages,” *Jpn. J. Appl. Phys.*, vol. 51, pp. 09MK05-1–09MK05-4, Sep. 2012.

[12] Z. Y. Liu, S. Liu, K. Wang, and X. B. Luo, “Studies on optical consistency of white LEDs affected by phosphor thickness and concentration using optical simulation,” *IEEE Trans. Compon. Packag. Technol.*, vol. 33, no. 4, pp. 680–687, Dec. 2010.

[13] Z. Y. Liu, C. Li, B. H. Yu, Y. H. Wang, and H. B. Niu, “Effects of YAG:Ce phosphor particle size on luminous flux and angular color uniformity of phosphor-converted white LEDs,” *J. Display Technol.*, vol. 8, no. 6, pp. 329–335, 2012.

[14] R. Hu, X. Fu, Y. Zou, and X. B. Luo, “A complementary study to ‘Toward scatter-free phosphors in white phosphor-converted light-emitting diodes’: Comment,” *Opt. Express*, vol. 21, no. 4, pp. 5071–5073, 2013.

[15] Y. H. Won, H. S. Jang, K. W. Cho, Y. S. Song, D. Y. Jeon, and H. K. Kwon, “Effect of phosphor geometry on the luminous efficiency of high-power white light-emitting diodes with excellent color rendering property,” *Opt. Lett.*, vol. 34, no. 1, pp. 1–3, 2009.

[16] R. Y. Yu, S. Z. Jin, S. Y. Cen, and P. Liang, “Effect of the phosphor geometry on the luminous flux of phosphor-converted light-emitting diodes,” *IEEE Photon. Technol. Lett.*, vol. 22, no. 23, pp. 1765–1767, Dec. 1, 2010.

[17] R. Hu, X. B. Luo, and S. Liu, “Effect of the amount of phosphor silicone gel on optical property of white light-emitting diodes packaging,” in *Proc. 12th ICEPT-HDP*, Shanghai, China, Aug. 2011, pp. 1081–1084.

[18] Z. Y. Liu, S. Liu, K. Wang, and X. B. Luo, “Optical analysis of color distribution in white LEDs with various packaging methods,” *IEEE Photon. Technol. Lett.*, vol. 20, no. 24, pp. 2027–2029, Dec. 15, 2008.

[19] Y. T. Zhu and N. Narendran, “Investigation of remote-phosphor white light-emitting diodes with multi-phosphor layers,” *Jpn. J. Appl. Phys.*, vol. 49, pp. 100203-1–100203-3, Oct. 2010.

[20] H. Luo, J. K. Kim, E. F. Schubert, J. Cho, C. Sone, and Y. Park, “Analysis of high-power packages for phosphor based white-light-emitting diodes,” *Appl. Phys. Lett.*, vol. 86, no. 24, pp. 243505–243507, 2005.

[21] H. K. Park, J. H. Oh, and Y. R. Do, “Toward scatter-free phosphors in white phosphor-converted light-emitting diodes,” *Opt. Express*, vol. 20, no. 9, pp. 10218–10228, 2012.

[22] J. P. You, N. T. Tran, and F. G. Shi, “Light extraction enhanced white light-emitting diodes with multi-layered phosphor configuration,” *Opt. Express*, vol. 18, no. 5, pp. 5055–5010, 2010.

[23] B. C. Li, D. W. Zhang, Y. S. Huang, Z. J. Ni, and S. L. Zhuang, “A new structure of multi-layer phosphor package of white LED with high efficiency,” *Chin. Opt. Lett.*, vol. 8, no. 2, pp. 221–223, 2010.

[24] R. Hu and X. B. Luo, “A model for calculating the bidirectional scattering properties of phosphor layer in white light-emitting diodes,” *J. Lightw. Technol.*, vol. 30, no. 21, pp. 3376–3380, Nov. 1, 2012.

[25] R. Hu, Y. M. Wang, Y. Zou, X. Chen, S. Liu, and X. B. Luo, “Study on phosphor sedimentation effect in white LED packages by modeling multi-layer phosphors with the modified Kubelka-Munk theory,” *J. Appl. Phys.*, vol. 113, pp. 063108-1–063108-6, 2013.

[26] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*. New York, NY, USA: Wiley, 1983, pp. 90–120.