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Carpet thermal cloak realization based on the refraction law of heat flux

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Abstract – Conventional studies on the design of a (carpet) thermal cloak are based on the coordinate transformation techniques, which are complicated and hard to realize with natural materials. Here we show another feasible approach to design the carpet thermal cloak through the rotated alternatively stacking materials based on the refraction law of heat flux in analogy to that of light. To make the consequent bending of heat flux parallel to the profile of the cloaking region, three design rules to realize the carpet cloaking effect by such configurations are provided and validated. The boundary condition tolerances of the carpet thermal cloak are examined as well.

Introduction. – Artificially engineered materials with specific thermal properties, known as thermal metamaterials, exhibit unusual thermal properties. They have been used to channel the propagation of heat flow and applied to cloaking through transformation thermodynamics which was proposed in analogy to transformation optics [1–4]. Under the coordinate transformation in transformation thermodynamics, the invariance of the diffusive equation allows the space around the object to be reshaped such that the heat can propagate in the desired way [1–5]. Based on this result, some new heat transfer phenomena, like shielding, concentrating or even inverting the heat flow, become possible, which were hard to imagine in the past [6,7]. Thermal cloak, as the typical outcome of transformation thermodynamics, can hide an object from heat or render heat invisible in a certain region [5,8–10]. Many studies related to the design/fabrication of a thermal cloak based on either thermal metamaterials or engineering materials exist in the literature [1,5–10]. On the other hand, a recently published theory has suggested an electromagnetic (EM) carpet cloak or ground-cloak, in which any objects hidden under the carpet appear as a flat conducting sheet [11–17]. The carpet cloak has triggered a boom of studies due to its superior properties in terms of inherently relaxed constraints, experimental feasibility, and wide application over conventional spherical invisible cloak. However, the attention that is paid to the counterpart of the EM carpet cloak, or called carpet thermal cloak, is not as great as that devoted to thermal cloak [18,19]. The extension of the EM carpet cloak into thermodynamics is thus of great significance. Moreover, conventional coordinate transformation is the prevalent method to design such cloaking structures, which is rather complicated and lacking consideration of fabrication feasibility [1,5,20–22].

Very recently, a major breakthrough that is not based on previous EM-related principles has been achieved with the aim of controlling heat flux propagation [23–28]. The thermal extremum principle, where the propagation of heat takes the path of least thermal resistance, yields quantitative criteria for tracing and consequently manipulating the propagation of heat flux, resulting in the origin of the refraction law of heat flux [26]. In this letter, we try to realize the carpet thermal cloak without utilizing the coordinate transformation techniques. Instead, we apply the refraction law of heat flux to bend the heat flow along the stacking thermal metamaterials, so as to realize the carpet thermal cloak.

Design methods. – According to the refraction law of heat flux, when heat flows through an interface with two materials of different thermal conductivities κ_1 and κ_2, as shown in fig. 1(a), the heat flux is demonstrated to

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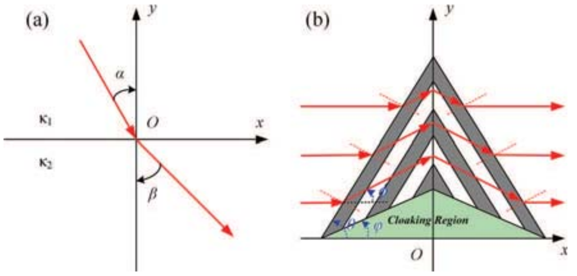

Fig. 1: (Colour on-line) (a) Schematic of the refraction of heat flux vectors at the interface of two materials with thermal conductivities κ_1 and κ_2, and (b) schematic of the trajectory of heat flow through a carpet thermal cloak made by rotated alternatively stacking materials.

propagate through the interface under a tangent law [25], *i.e.*,

$$\frac{\tan\alpha}{\kappa_1} = \frac{\tan\beta}{\kappa_2}, \tag{1}$$

where α is the incidence angle, and β is the refraction angle. When both the neighboring materials are isotropic, the heat flux q_i (from Fourier's law of conduction, heat transport is diffusive with the flux in the i-th direction $q_i = -\kappa_{ij}\nabla T_j$, where κ_{ij} represents the components of the second-order thermal conductivity tensor, and ∇T_j represents the temperature gradient in the j-th direction.) follows the respective temperature gradient, namely the heat flux along the x-axis is only determined by the temperature gradient in that direction. When the neighboring materials are anisotropic, *i.e.* the off-diagonal terms (κ_{ij} with $i \neq j$) are not null, cross-coupling and concomitant bending of the heat flux occurs, *e.g.* with a substantial κ_{xy}, the heat flux along the x-axis would be determined by the temperature gradient in both the x- and an orthogonal direction. It has been pointed out [23] that stacking materials, *e.g.* alternatively stacked two sheets with different thermal conductivities, would enable the control of the bending/manipulation of the thermal flux and have the advantages of easy fabrication over the ideal metamaterials whose anisotropic distribution of thermal conductivity is hard to realize though they have wonderful cloaking/bending properties. For a rotated composite (say, by an angle of θ) of two alternating materials with isotropic thermal conductivity (κ_1 and κ_2, say, with $\kappa_1 > \kappa_2$), with respect to a horizontal thermal gradient direction, the bending angle of heat flux ϕ can be calculated based on the extension of the refraction law [23], as

$$\phi = \tan^{-1}\left[\frac{(c-1)\cos\theta\sin\theta}{\cos^2\theta + c\sin^2\theta}\right] \quad \text{with} \quad c = \frac{(1+\kappa_1/\kappa_2)^2}{4\kappa_1/\kappa_2}. \tag{2}$$

As shown in fig. 1(b), the carpet cloak, assembled by alternative two stacking isotropic materials, is rotated by θ counterclockwise with respect to the x-axis in the second quadrant and is symmetrical to the y-axis. The cloaking region is a triangle that plays the role of "bump" on a flat ground with a tilt angle φ. Heat flows along the x-axis from left to right with incidence angle of $(\pi/2-\theta)$ with respect to the normal of the rotated carpet cloak. Theoretically, any arbitrary object we aim to conceal from outside the heat flux can be placed inside the bump (cloaking region), as long as it fits in the volume. With eq. (2), we can calculate the bending angle of the heat flux ϕ across the stacking materials. To realize the function of the carpet thermal cloak, we could tune the heat flux and make the heat flux return to its original direction after passing through the cloak by adjusting the rotation angle θ and stacking materials. For example, when the tilt angle φ equals the bending angle ϕ, the heat flux vector will be parallel to the profile of the cloaking region. Due to the symmetry, the heat flux vector will return to its original horizontal direction in the end, and it seems as if there is no bump or hidden object on the "flat" ground. Moreover, it is seen from fig. 1(b) that the rotation angle should be larger than the tilt angle; otherwise the cloaking structure is covered by the bump, which will violate the original intention of the carpet thermal cloak. Thus, we have $\theta > \phi$, by substituting into eq. (2), yielding

$$\begin{aligned}\theta &> \tan^{-1}\sqrt{\frac{c-2}{c}} \\ &= \tan^{-1}\left(\frac{\sqrt{(\kappa_1/\kappa_2)^2 - 6(\kappa_1/\kappa_2)+1}}{\kappa_1/\kappa_2+1}\right).\end{aligned} \tag{3}$$

The right-hand term in eq. (3) is denoted as the critical value of the rotation angle (θ_{cr}) here and this is the first rule for which the rotation angle should be larger than the critical value. To make sense of θ_{cr}, the term $(\kappa_1/\kappa_2)^2 - 6(\kappa_1/\kappa_2) + 1 > 0$, then $\kappa_1/\kappa_2 > 5.83$. This is another rule to select the proper stacking materials to realize the present carpet thermal cloak.

According to eq. (2), the rotation angle θ could be rewritten as the parabolic function of φ and κ_1/κ_2 where the bending angle ϕ is assumed to be the same as the tilt angle φ. To make sense of the root of the parabolic equation of θ, we can derive

$$\varphi < \tan^{-1}\left(\frac{c-1}{2\sqrt{c}}\right) = \tan^{-1}\left(\frac{(\kappa_1/\kappa_2-1)^2}{4(\kappa_1/\kappa_2+1)\sqrt{\kappa_1/\kappa_2}}\right). \tag{4}$$

The right-hand term in eq. (4) is denoted as the critical value of the tilt angle (φ_{cr}) here and this is the rule to quantify the tilt angle of the cloaking region. It is perceived that a larger tilt angle leads to a larger cloaking region. From eqs. (3) and (4), θ_{cr} and φ_{cr} are functions of κ_1/κ_2, and the corresponding variations are shown in fig. 2 when κ_1/κ_2 increases from 10 to 100. It is seen that θ_{cr} and φ_{cr} are linearly dependent on κ_1/κ_2, and θ_{cr} is less than φ_{cr}. Since θ should be larger than θ_{cr} and φ should be less than φ_{cr} according to eqs. (3) and (4), the yellow region above the red line could be chosen as the domain of the rotation angle, and the blue hatched region below

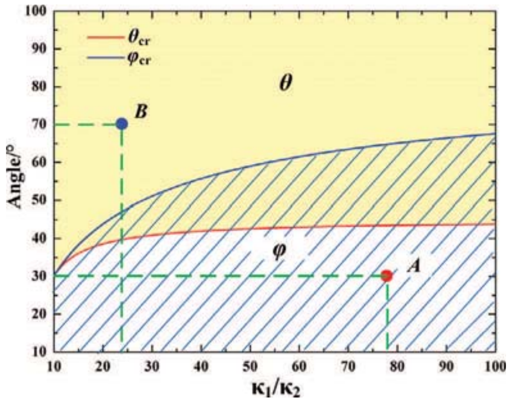

Fig. 2: (Colour on-line) Variation of the critical rotation angle $\theta_{\rm cr}$ and critical tilt angle $\varphi_{\rm cr}$ *vs.* κ_1/κ_2.

the blue line could be chosen as the domain of the tilt angle. And we must bear in mind that the rotation angle should be larger than the tilt angle, so we should check the constraints in eqs. (3) and (4) in each design case.

Results and discussions. – To illustrate the performance of the proposed carpet thermal cloak, we performed finite element simulations with commercial software package (COMSOL MULTIPHYSICS). In our simulation, the stacking sheets whose thickness is 1 mm are assumed to be in contact tightly with each other and thus the interfacial thermal resistance could be neglected. The whole domain dimensions are 200 mm × 120 mm. The left and right boundaries are kept at 400 K and 300 K, respectively. The thermal conductivities of the ambient media and the cloaking region are 1 and 10 W/mK, respectively. The simulated temperature profiles of the carpet thermal cloak made by different materials are shown in fig. 3.

In fig. 3(a), the stacking materials are as homogenous as the ambient media. In fig. 3(b), copper ($\kappa = 390\,$W/mK) and thermally conducting silver epoxy ($\kappa = 5\,$W/mK) are used to fabricate the cloaking structure. The critical tilt angle is calculated as 64°, thus we assign the tilt angle as 30° to design the cloaking structure, as point A in fig. 2. With eq. (2), the rotation angle can be calculated as 58.2°, which is larger than the critical rotation angle of 43.5°. In fig. 2(c), brass ($\kappa = 120\,$W/mK) and epoxy are used as stacking materials, where $\theta_{\rm cr}$ and $\varphi_{\rm cr}$ are 39.8° and 47.2°, respectively. In this case, we assign the rotation angle as 70° to design the cloaking structure, as point B in fig. 2, and thus the corresponding tilt angle is 32.2°. The corresponding temperature fields of each case are illustrated in fig. 3, where the green lines show the direction of the heat flow. It is seen in fig. 3(a) that more heat tends to propagate in the cloaking region since the thermal conductivity there is larger. But in figs. 3(b) and (c), heat diffuses uniformly in the beginning; when reaching the cloak, heat tends to bend upward along the profile of the cloaking region without passing through the cloaking region, just as the design in which the bending angle equals the tilt angle of the cloaking region; after passing through the cloak, heat gradually returns to the original horizontal direction in the end. Compared with fig. 3(a), the cloaking effect in figs. 3(b) and (c) is obvious, which is consistent with our original purpose of the cloaking design. Therefore, it is by designing the rotated stacking materials with the consideration of rotation angle, tilt angle and thermal conductivity ratio simultaneously that we can realize the carpet thermal cloaking effect, rather than employing the relatively complicated coordinate transformation techniques. We can either design the rotation angle by the tilt angle (fig. 3(b)) or design the tilt angle by the rotation angle (fig. 3(c)) with proper materials. Both approaches are demonstrated to be effective to realize the carpet thermal cloak.

The boundary condition tolerances of the carpet thermal cloak are examined by rotating the carpet thermal

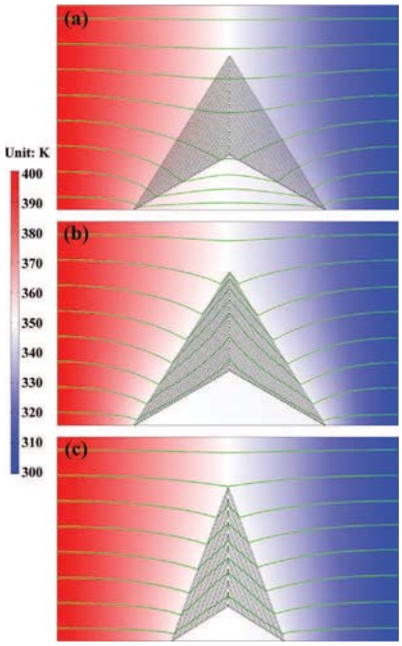

Fig. 3: (Colour on-line) Temperature profile comparison among the carpet thermal cloak made by different materials: (a) homogeneous materials, (b) copper and epoxy, and (c) brass and epoxy.

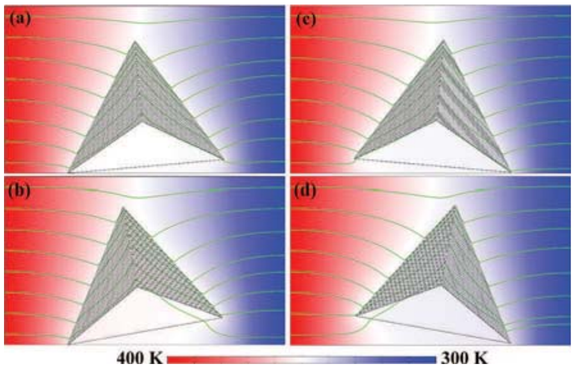

Fig. 4: (Colour on-line) Boundary condition tolerance in the temperature profile of the carpet thermal cloak with different rotations: (a) +5°, (b) +10°, (c) −5°, and (d) −10°.

cloak with respect to the x-axis. The corresponding temperature profiles are illustrated in fig. 4. From figs. 4(a) and (c), though the carpet thermal cloaks are rotated by ±5° respectively, the heat flow still preserves its diffusive characteristics, *i.e.*, heat bends upward along the profile of the cloaking region without passing through the cloaking region. But the situations change when the cloaks are rotated further. From figs. 4(b) and (d), when the carpet thermal cloaks are rotated by ±10°, respectively, part of the heat flow no longer preserves its previous behaviour, *i.e.*, heat passes through the cloaking region, although most heat still diffuses along the profile of the cloaking region.

The reasons behind this phenomenon can be explained as follows. Firstly, the carpet thermal cloak is designed for the case in which heat diffuses along the x-axis, and the structure of the present carpet thermal cloak is merely symmetrical along the x-axis. Therefore, when the boundary conditions are not symmetrical along the x-axis, the thermal cloaking effect will be greatly influenced and the boundary condition tolerance is challenged. Secondly, from the refraction law of heat flux in eq. (1), the refraction angle is dependent on the incident angle of heat flux, and the bending angle (difference of the refraction angle and the incident angle) is the primary parameter to design the carpet thermal cloak. Once the carpet thermal cloak is given, any perturbation of the incident angle will make a difference on the refraction/bending angle, thus the cloaking effect is challenged. This intrinsic dependence on the boundary conditions is derived from the refraction law of heat flux, which is the exact limitation of such carpet thermal cloak configuration.

Conclusions. – In summary, our work indicates that a carpet thermal cloak can be realized by the rotated stacking materials, without employing the conventional coordinate transformation techniques. Three rules to realize the carpet cloaking effect by such structure are provided, and both the critical rotation angle and critical tilt angle are found to be functions of the thermal conductivity ratio (κ_1/κ_2). We can realize the carpet thermal cloak by designing the rotation angle or the tilt angle with proper materials for the stacking structure. The boundary condition tolerances of the carpet thermal cloak are examined. The implementation of the proposed carpet thermal cloak is benefited from the refraction law of heat flux, which provides alternative ways to control/tune the heat flow for directing/harvesting the thermal energy with useful purposes.

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