



# Calculation of the phosphor heat generation in phosphor-converted light-emitting diodes



Xiaobing Luo\*, Run Hu<sup>1</sup>

School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China

## ARTICLE INFO

### Article history:

Received 25 April 2013

Received in revised form 9 January 2014

Accepted 24 March 2014

### Keywords:

Phosphor heat generation

Light-emitting diode

Kubelka–Munk theory

## ABSTRACT

Phosphor heat generation in phosphor-converted light-emitting diodes (pc-LEDs) plays an important role in affecting the optical and thermal performance of LEDs, but it is hard to measure directly. In this paper, we developed a method to calculate the phosphor heat generation by modifying the Kubelka–Munk theory. With full consideration of light scattering, absorption and conversion inside the phosphor layer simultaneously, we calculated the phosphor heat generation and analyzed its trend with changing the phosphor quantum efficiency and phosphor parameters (concentration, thickness, particle size). Experiments were also conducted to validate the calculations. It was found that with the increase of phosphor concentration from 0.11 to 0.34 g/cm<sup>3</sup>, the total energy loss increases by 54.11%. The phosphor concentration and thickness played similar roles in affecting the phosphor heat generation. The maximum phosphor heat generation corresponded to the phosphor particle size of around 2 μm, which is denoted as the critical size.

© 2014 Elsevier Ltd. All rights reserved.

## 1. Introduction

Benefited by their extraordinary features over traditional light sources, high power white light-emitting diodes (LEDs) have been extensively used in many areas from the indoor illumination to the outdoor lighting and landscaping, from the backlighting in the personal digital assistant (PDA) to the large-scale TV sets, from the vehicle headlamp to the airborne or cabin lighting, etc [1–7]. Moreover, the luminous efficiency, robustness, and lifetime of LEDs are still under rapid development. Therefore, white LEDs have been regarded as the fourth generation of light source. According to the current state-of-the-art white LEDs, there are three frequently-used approaches to generate white light based on LEDs. The first one is based on a blue LED chip coated with yellow phosphors. The second one is based on ultraviolet LED chip coated with blue and yellow phosphors (or red, blue and yellow phosphors). The third one is a device that combines red, green and blue LED chips together [8]. Among these approaches, the former two methods are involved with phosphor materials. As one kind of the photoluminescence materials, phosphors in phosphor-converted LEDs (pc-LEDs) absorb part of short-wavelength light emitted from LED chip and convert part of the absorbed short-wavelength light into long-

wavelength light. The energy ratio between the emitted light and the absorbed light, which is called as the energy conversion efficiency  $\eta_{con}$ , is determined by the quantum efficiency  $\eta_{QE}$  of phosphors, as expressed in Eq. (1).

$$\eta_{con} = \frac{N_{emitted}}{N_{absorbed}} \frac{hc/\lambda_{emitted}}{hc/\lambda_{absorbed}} = \eta_{QE} \frac{\lambda_{absorbed}}{\lambda_{emitted}} \quad (1)$$

Due to the non-radiative relaxation processes in the 4f- or 5d- energy level of the doping ions in the down-converting phosphors,  $\eta_{con}$  is usually below unit [9]. The rest of the absorbed light is converted to heat consequently. Moreover, in the pc-LED packaging, the phosphors are usually dispersed in the silicone with low thermal conductivity, so the heat generated in phosphors is hard to dissipate [10]. As a result, the phosphor temperature increases greatly. High phosphor temperature will decrease its quantum efficiency exponentially, induce the local stress and even delamination, and degrade the reliability and lifetime, etc [11–14]. Therefore, people are trying to obtain the phosphor temperature or phosphor heat generation. It has been reported that the local phosphor temperature becomes the highest in the LED packages rather than the junction temperature in some situations [15–18]. In our previous study [19], we discovered that the hotspot in the LED packages would shift to the phosphor layer when varying the phosphor coating positions or concentrations. We even observed the shocking silicone carbonization phenomenon caused by the effect of phosphor self-heating in multi-chip array LED packaging [20]. However, since

\* Corresponding author. Tel.: +86 13971460283; fax: +86 27 87540724.

E-mail addresses: [luoxb@hust.edu.cn](mailto:luoxb@hust.edu.cn) (X. Luo), [hurun@hust.edu.cn](mailto:hurun@hust.edu.cn) (R. Hu).

<sup>1</sup> Tel.: +86 27 87542604; fax: +86 27 87557074.

## Nomenclature

$A$	undetermined coefficient
$a$	absorption coefficient of phosphor, $\text{mm}^{-1}$
$B$	undetermined coefficient
$C$	undetermined coefficient
$c$	light speed, m/s
$D$	undetermined coefficient
$d$	phosphor thickness, $\mu\text{m}$
$E_0$	incident light energy, W
$E(z)$	forward scattering light energy function, W
$e$	natural base number
$F(z)$	backscattering light energy function, W
$h$	Planck constant, $\text{m}^2\text{kg/s}$
$N_{\text{absorbed}}$	number of absorbed photons
$N_{\text{emitted}}$	number of emitted photons
$P_B$	optical power of blue LED, W
$P_W$	optical power of white LED, W
$Q_{\text{phos}}(z)$	phosphor heat generation function, W
$Q_{\text{phos\_total}}$	total phosphor heat generation, W

$Q_{\text{ref}}$	reflecting light energy loss, W
$Q_{\text{total}}$	total energy loss, W
$s$	scattering coefficient of phosphor, $\text{mm}^{-1}$
$z$	invasion depth, $\mu\text{m}$

## Greek Symbols

$\alpha$	$\alpha = 2\sqrt{a_B(a_B + 2s_B)}$
$\beta$	$\beta = \sqrt{a_B/(a_B + 2s_B)}$
$\gamma$	reflection coefficient
$\eta_{\text{con}}$	energy conversion efficiency
$\eta_{\text{QE}}$	phosphor quantum efficiency
$\lambda_{\text{absorbed}}$	absorbed wavelength, nm
$\lambda_{\text{emitted}}$	emitted wavelength, nm
$\mu$	$\mu = 2\sqrt{a_Y(a_Y + 2s_Y)}$
$\nu$	$\nu = \sqrt{a_Y/(a_Y + 2s_Y)}$

## Subscript

$B$	blue light
$Y$	yellow light

the phosphor particles are dispersed in the encapsulants (silicone or epoxy), it is difficult to measure the phosphor temperature directly. So, most of the aforementioned work was based on simulations, including Monte Carlo ray-tracing method and finite element method. In these simulations, the phosphor heat generations were assigned by some simplifications or assumptions. As far as we know, there are few systematic works on the calculation of phosphor heat generation in theory.

In this study, we developed a method to calculate the phosphor heat generation by revising the Kubelka–Munk theory. Light scattering, absorption and conversion were considered simultaneously to close to the real light propagation. The phosphor heat generations were calculated with changing the quantum efficiency based on our method. The influences of the phosphor parameters (thickness, concentration, particle size distribution) were analyzed. Experiments were also conducted to validate the calculations.

## 2. Calculation method

In our previous study, we modified the Kubelka–Munk theory by considering the light conversion process in light propagation in phosphor-embedded silicone matrix [21]. As shown in Fig. 1, for a thin phosphor layer, the forward scattering light energy  $E(z)$

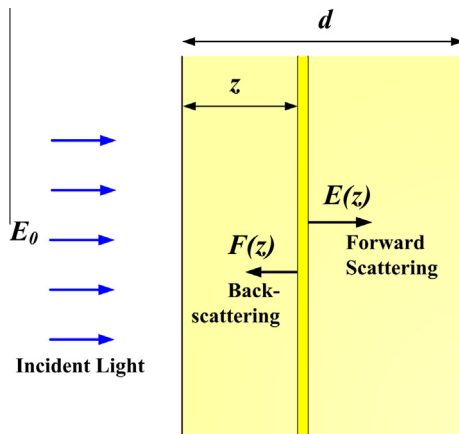


Fig. 1. Forward scattering and backscattering functions with invasion depth  $z$  in a phosphor layer.

and backscattering light energy  $F(z)$  for both blue light and yellow light can be expressed as

$$dE_B(z)/dz = -2(a_B + s_B)E_B(z) + 2s_B F_B(z) \quad (2)$$

$$dF_B(z)/dz = 2(a_B + s_B)F_B(z) - 2s_B E_B(z) \quad (3)$$

$$dE_Y(z)/dz = -2(a_Y + s_Y)E_Y(z) + 2s_Y F_Y(z) + \eta_{\text{con}} a_B [E_B(z) + F_B(z)] \quad (4)$$

$$dF_Y(z)/dz = 2(a_Y + s_Y)F_Y(z) - 2s_Y E_Y(z) - \eta_{\text{con}} a_B [E_B(z) + F_B(z)] \quad (5)$$

The appearance of coefficient 2 in these equations is because that the mean path of light inside the isotropic phosphor layer is double of the thickness [22]. The necessary scattering and absorption coefficients of phosphors, which vary with the change of phosphor concentration and particle size, are calculated in advance according to Mie–Lorenz theory [23]. By solving Eqs. (2)–(5), we could obtain the general forms of forward scattering light energy  $E(z)$  and backscattering light energy  $F(z)$  as Eqs. (6) and (7) for both blue light and yellow light, respectively.

For blue light:

$$\begin{aligned} E_B(z) &= A(1 - \beta)e^{2z} + B(1 + \beta)e^{-2z} \\ F_B(z) &= A(1 + \beta)e^{2z} + B(1 - \beta)e^{-2z} \end{aligned} \quad (6)$$

For yellow light:

$$\begin{aligned} E_Y(z) &= C(1 - \nu)e^{\mu z} + D(1 + \nu)e^{-\mu z} \\ &\quad + \frac{2\eta_{\text{con}} a_B}{\nu(\mu^2 - 2^2)} [A(\mu - \nu\alpha)e^{2z} + B(\mu + \nu\alpha)e^{-2z}] \\ F_Y(z) &= C(1 + \nu)e^{\mu z} + D(1 - \nu)e^{-\mu z} \\ &\quad + \frac{2\eta_{\text{con}} a_B}{\nu(\mu^2 - 2^2)} [A(\mu + \nu\alpha)e^{2z} + B(\mu - \nu\alpha)e^{-2z}] \end{aligned} \quad (7)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are the coefficients. To solve these coefficients, we have to balance the numbers of unknowns and equations with proper boundary conditions. For the phosphor layer coated on LED chips, the plane  $z = 0$  means the interface between the LED chip and the phosphor layer. Therefore, the backscattered blue and yellow light projected at the interface will be reflected back to the phosphor layer. The boundary conditions for blue light and yellow light are:

$$E_B(0) = E_0 + \gamma_B F_B(0), \quad F_B(d) = 0 \quad (8)$$

$$E_Y(0) = \gamma_Y F_Y(0), \quad F_Y(d) = 0 \quad (9)$$

Until now, we can obtain the forward scattering and backscattering light energy along with the invasion depth  $z$ . The phosphor heat generation  $Q_{phos}(z)$  could be calculated as

$$Q_{phos}(z) = (1 - \eta_{con})a_B[E_B(z) + F_B(z)] + a_Y[E_Y(z) + F_Y(z)] \quad (10)$$

Substituting with Eqs. (6) and (7) into Eq. (10), we can obtain the phosphor heat generation as the function of the invasion depth  $z$ , as expressed in Eq. (11).

$$Q_{phos}(z) = 2(1 - \eta_{con})a_B[Ae^{zz} + Be^{-zz}] + 2a_Y \left[ Ce^{\mu z} + De^{-\mu z} + \frac{2\eta_{con}a_B\mu}{v(\mu^2 - \alpha^2)}(Ae^{zz} + Be^{-zz}) \right] \quad (11)$$

In the right side of Eq. (11), the first term is the absorption of blue light, and the second term is the absorption of yellow light. The energy losses in white LED include two parts: one is the phosphor heat generation, and the other one is the light reflection loss by the LED chips or substrate. The total heat generated in phosphors could be calculated by integrating the heat generation function  $Q_{phos}(z)$  along the phosphor thickness, as shown in Eq. (12). The light reflection loss can be calculated by Eq. (13).

$$Q_{phos\_total} = \int_0^d Q_{phos}(z) dz \quad (12)$$

$$Q_{ref} = (1 - \gamma_B)F_B(0) + (1 - \gamma_Y)F_Y(0) \quad (13)$$

The phosphor heat generation function  $Q_{phos}(z)$ , the total heat generation  $Q_{phos\_total}$ , and the reflection loss  $Q_{ref}$  can be normalized by dividing the input light energy  $E_0$ . Therefore, the total energy loss in white LEDs can be determined as

$$Q_{total} = Q_{phos\_total} + Q_{ref} \quad (14)$$

### 3. Experiment

To validate the calculations of phosphor heat generation in white LEDs, experiments were also conducted. Three white LEDs with different phosphor concentrations and one blue LED without phosphors were tested. All the LED chips are the same except the phosphor concentration. The input current was kept as 350 mA. The correlated color temperature (CCT) and optical power of the LEDs were measured by an integrated sphere. Obviously, the optical power of blue LED without phosphors is the biggest. Assuming that the light extraction efficiencies (LEEs) of blue LED and white LEDs are the same and neglecting the light absorption in encapsulants and optical lens, the total energy losses, which include the phosphor heat generation and light reflection loss, can be calculated as the optical power difference between the optical power of blue LED and white LEDs, as shown in Eq. (15).

$$Q_{total} = P_B - P_W \quad (15)$$

### 4. Results and discussions

#### 4.1. Phosphor heat generation function

According to Eq. (11), we obtained the phosphor heat generation function  $Q_{phos}(z)$ , which was normalized and plotted along the invasion depth  $z$  in Fig. 2. The area below the phosphor heat generation curve is the normalized total phosphor heat generation. It is seen that phosphor heat generation decreases prominently with the increase of invasion depth. It implies that when the phosphors are close to the LED chip (invasion depth is small), the phosphors generate more heat; when the phosphors are far away from the LED chip (invasion depth is large), the phosphors generate less heat. This phenomenon is due to that more light scattering,

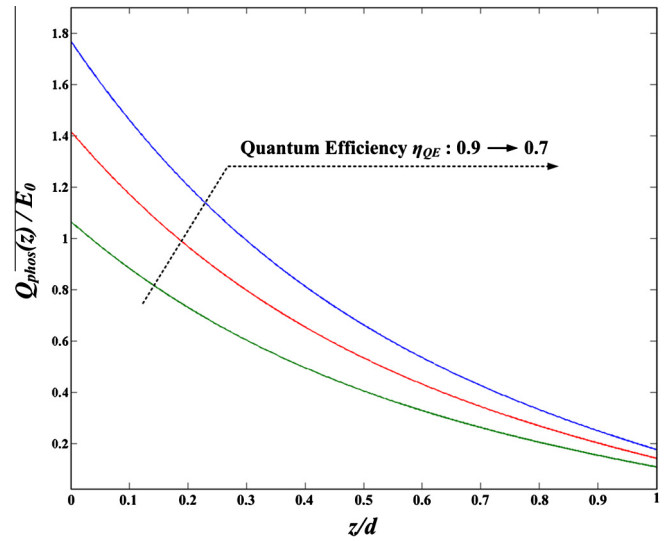


Fig. 2. Normalized phosphor heat generation function with changing quantum efficiencies.

absorption and conversion events happen when the phosphors are close to the chip. From Fig. 2, we also can see that when the quantum efficiency increases from 0.7 to 0.9, the phosphor heat generation decreases. According to Eq. (11), we can conclude that when the quantum efficiency increases, the deduction of heat generation caused by blue light absorption overwhelms the enhancement of heat generation caused by yellow light absorption. In the phosphor heat generation, the heat generation caused by blue light absorption accounts for larger margin. The phenomenon is easy to understand: when the quantum efficiency increases, more absorbed blue light is converted to yellow emission, thus less heat is generated. Therefore, for the phosphors in LED packaging, people try to enhance the quantum efficiency by different approaches, such as surface treatment, post-synthesis annealing, etc [24,25].

#### 4.2. Comparisons with experiments

The experimental results of the four LEDs were listed in Table 1. From the data in Table 1, we can see that with the increase of phosphor concentration, the CCT and optical power decrease greatly. The decrease denote as the transition from cool white light to warm white light. According to Eq. (15), the total energy loss was calculated. To validate the present method, we plotted the calculations and experimental results together. As shown in Fig. 3, the experimental results agreed with our calculations. With the increase of phosphor concentration, the total heat generation becomes larger both in the experimental and calculation results. The reasons behind the deviations between the experimental results and model predictions are: (1) in the model, the incident rays are vertical while it isn't exactly so in the experiments; (2) the reflection coefficients of blue light and yellow light may be different in the experiments; (3) the light energy losses absorbed by the encapsulants and optical lens were neglected in the experiments. It is seen in the experiments that with the increase of phosphor con-

Table 1  
Experimental results.

	Blue LED	LED 1	LED 2	LED 3
Concentration [g/cm <sup>3</sup> ]	0	0.11	0.21	0.34
CCT [K]	–	8816	4360	3735
Optical Power [mW]	467	321	286	242

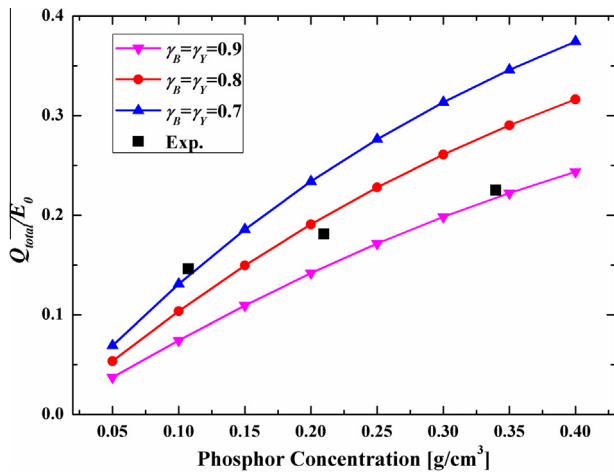


Fig. 3. Comparison of total energy losses obtained by experiments and calculations.

centration from 0.11 to 0.34 g/cm<sup>3</sup>, the total energy loss increases by 54.11%. The enhancement of reflection coefficient results in the decrease of total energy loss, which is easy to understand according to Eq. (13). When the reflection coefficient is small, more light would be absorbed directly by the surface. When the reflection coefficient is large, more light energy would be reflected into the phosphor layer, and more light energy would be transmitted through the layer. Surface treatment, like coating silver, therefore, is a good way to enhance the reflection coefficient and decrease the energy loss. We also can notice that in warm light LED, more energy would be lost and more heat would be generated inside the package.

4.3. Effect of phosphor parameters

According to Mie–Lorenz theory, the phosphor parameters (concentration, thickness, and particle size) play significant roles in determining the scattering and absorption coefficients of phosphors [23,26–28]. And these two coefficients are key parameters in the present method. Hence, it is necessary to study the effect of phosphor parameters on the phosphor heat generation.

The influences of phosphor concentration and thickness on the normalized total phosphor heat generation were illustrated in Fig. 4. In our calculations, the phosphor concentration varies from 0.05 to 1.0 g/cm<sup>3</sup>, and phosphor thickness varies from 0.1 to 0.3 mm. It is seen that the total phosphor heat generation increases

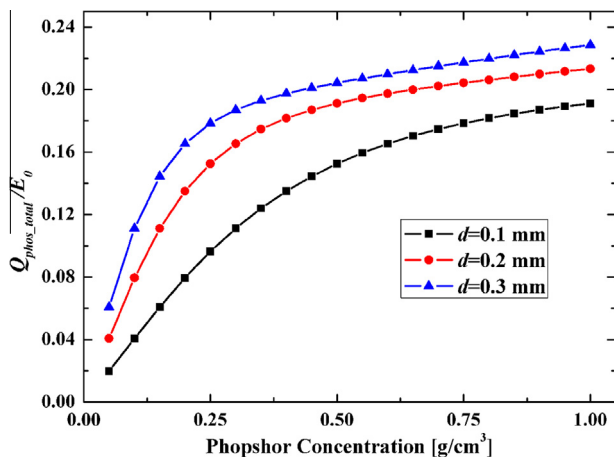


Fig. 4. Total phosphor heat generation with different concentrations and thicknesses.

with the increase of phosphor concentration or thickness. When the phosphor concentration varies from 0.05 to 1.0 g/cm<sup>3</sup>, the normalized total phosphor heat generations increase by 861.3%, 424.7%, and 275.9% under the phosphor thickness of 0.1, 0.2, and 0.3 mm, respectively. It also can be seen that with the increase of phosphor concentration, the slopes of the curves decreases gradually and the initial slopes are the largest. It implies that the when phosphor concentration is low, the effect of concentration is large; when the concentration is large, the effect is small. When the thickness increases, the initial slope increases, because for larger thickness, a small increase in concentration leads to considerable enhancement of phosphor amount, resulting in larger phosphor heat generation. It is concluded that the phosphor concentration and thickness play a similar role in affecting the phosphor heat generation. This can be understood that the both phosphor concentration and thickness determine the phosphor amount. The larger phosphor amount is, the more light scattering, absorption and conversion events happen, and the more heat would be generated.

The effects of phosphor particle size and quantum efficiency on the normalized total phosphor heat generation were investigated and the trends were plotted in Fig. 5. It is seen that with the increase of quantum efficiency, the total phosphor heat generation decreases. According to definition of quantum efficiency in Eq. (1), the larger quantum efficiency, the more converted photons can be emitted, thus less photons are trapped and converted to heat. From Fig. 5, we can see that the total phosphor heat generation increases in the initial stage. The peaks in the curves correspond to the critical sizes, which are about 2 μm. When the particle size becomes larger than the critical size, the total phosphor heat generation decreases. The reasons behind these phenomena lie in the change of scattering coefficient with the phosphor particle size. Park et al. [29] has pointed out that the scattering coefficient increases at first and decreases subsequently. When the particles are small, most of the incident light would penetrate the phosphor layer without scattering, absorption and conversion, thus the heat generation is small. When the particles approach the critical size, the light would be scattered in all the directions homogeneously and the scattering events grow in number, and the heat generation increases. Therefore, when the particle size is smaller than the critical size, the total phosphor heat generation increases with the increase of particle size. When the particles become larger than the critical size, most of the incident light would be forwardly scattered, thus the number of light scattering events decreases. Therefore, the total phosphor heat generation decreases with the increase of particle size when the particle size becomes larger than the critical size.

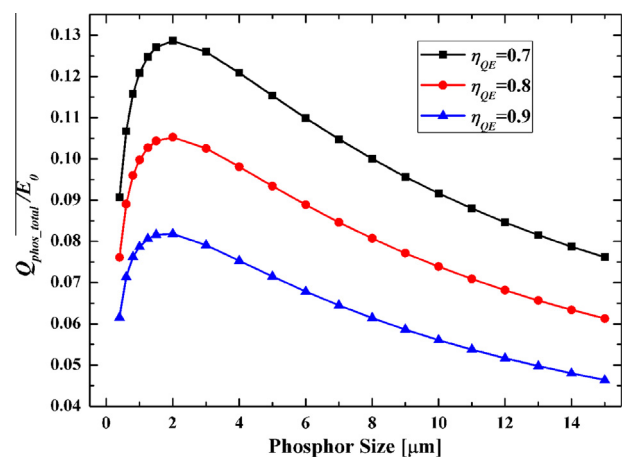


Fig. 5. Total phosphor heat generation with different phosphor particle sizes and quantum efficiencies.

## 5. Conclusions

In this paper, we presented a method to calculate the phosphor heat generation in white light-emitting diode (LED) packages. The present method was developed by modifying the Kubelka–Munk theory with full consideration of light scattering, absorption, and conversion simultaneously. Experiments were conducted to validate the method and the experimental results agreed well with the theoretical predictions. It was found in the experiments that with the increase of phosphor concentration from 0.11 to 0.34 g/cm<sup>3</sup>, the total energy loss increases by 54.11%. Based on the method, the effects of phosphor parameters (concentration, thickness, particle size, and quantum efficiency) on the total phosphor heat generation were obtained. The phosphor concentration and thickness played similar roles in affecting the phosphor heat generation. The maximum phosphor heat generation corresponded to the phosphor particle size of around 2 μm, which is denoted as the critical size.

## Conflict of Interest

None declared.

## Acknowledgements

The authors would like to acknowledge the financial support partly by National Science Foundation of China (51376070), and partly by 973 Project of The Ministry of Science and Technology of China (2011CB013105).

## References

- [1] S. Liu, X.B. Luo, LED Packaging for Lighting Applications: Design, Manufacturing and Testing, John Wiley & Sons, Singapore, 2011. Chap. 1.
- [2] E.F. Schubert, Light-Emitting Diodes, second ed., Cambridge, New York, 2006. Chap. 1.
- [3] E.F. Schubert, J.K. Kim, Solid-state light sources getting smart, *Science* 308 (2005) 1274–1279.
- [4] S. Tonzani, Time to change the bulb, *Nature* 459 (2009) 312–314.
- [5] R. Hu, X.B. Luo, H. Zheng, Z. Qin, Z.Q. Gan, B.L. Wu, S. Liu, Design of a novel freeform lens for LED uniform illumination and conformal phosphor coating, *Opt. Express* 20 (13) (2012) 13727–13737.
- [6] R. Hu, X.B. Luo, H. Feng, S. Liu, Effect of phosphor settling on the optical performance of phosphor-converted white light-emitting diodes, *J. Lumin.* 132 (2012) 1252–1256.
- [7] R. Hu, X.B. Luo, S. Liu, Study on the optical properties of conformal coating light-emitting diode by Monte Carlo simulation, *IEEE Photonics Technol. Lett.* 23 (20) (2011) 1673–1675.
- [8] R. Hu, S. Yu, Y. Zou, H. Zheng, F. Wang, S. Liu, X.B. Luo, Near-/mid-field effect of color mixing for single phosphor-converted light-emitting diode package, *IEEE Photonics Technol. Lett.* 25 (3) (2013) 246–249.
- [9] J. Andriessen, E. van der Kolk, P. Dorenbos, Lattice relaxation study of the 4f–5d excitation of Ce<sup>3+</sup>-doped LaCl<sub>3</sub>, LaBr<sub>3</sub>, and NaLaF<sub>4</sub>: Stokes shift by pseudo Jahn–Teller effect, *Phys. Rev. B* 76 (2007) 075124.
- [10] C. Yuan, X.B. Luo, A unit cell approach to compute thermal conductivity of uncured silicone/phosphor composites, *Int. J. Heat Mass Transfer* 56 (2013) 206–211.
- [11] M. Zachau, D. Becker, D. Berben, T. Fiedler, F. Jermann, F. Zwaschka, Phosphors for solid state lighting, *Proc. SPIE* 6910 (2008) 691010.
- [12] P.F. Smet, A.B. Parmentier, D. Poelman, Selecting conversion phosphors for white light-emitting diodes, *J. Electrochem. Soc.* 158 (6) (2011) R37–R54.
- [13] S. Ye, F. Xiao, Y.X. Pan, Y.Y. Ma, Q.Y. Zhang, Phosphors in phosphor-converted white light-emitting diodes: recent advances in materials, techniques and properties, *Mater. Sci. Eng. Rep.* 71 (2010) 1–34.
- [14] M.N. Tsvetkova, V.G. Korsakov, M.M. Sychev, B.V. Chernovets, G.V. Itkinson, Study of photophosphors for white LEDs, *J. Opt. Technol.* 78 (6) (2011) 403–407.
- [15] B.H. Yan, N.T. Tran, J.P. You, F.G. Shi, Can junction temperature alone characterize thermal performance of white LED emitters, *IEEE Photonics Technol. Lett.* 23 (9) (2011) 555–557.
- [16] M. Arik, S. Weaver, C. Becher, M. Hsing, and A. Srivastava, Effects of localized heat generations due to the color conversion in phosphor particles and layers of high brightness light emitting diodes, Proceedings of International Electronic Packaging Technical Conference and Exhibition, Maui, Hawaii, USA, 2003, 35015.
- [17] B.F. Fan, H. Wu, Y. Zhao, Y.L. Xian, Gang Wang, Study of phosphor thermal-isolated packaging technologies for high-power white light-emitting diodes, *IEEE Photonics Technol. Lett.* 19 (15) (2007) 1121–1123.
- [18] J.H. Hwang, Y.D. Kim, J.W. Kim, S.J. Jung, H.K. Kwon, T.H. Oh, Study on the effect of the relative position of the phosphor layer in the LED package on the high power LED lifetime, *Phys. Status Solidi C* 7 (7–8) (2010) 2157–2161.
- [19] R. Hu, X.B. Luo, H. Zheng, Hotspot location shift in the high-power phosphor-converted white light-emitting diode packages, *Jpn. J. Appl. Phys.* 51 (2012) 09MK05.
- [20] X.B. Luo, X. Fu, F. Chen, H. Zheng, Phosphor self-heating in phosphor converted light emitting diode packaging, *Int. J. Heat Mass Transfer* 58 (2013) 276–281.
- [21] R. Hu, X.B. Luo, A model for calculating the bidirectional scattering properties of phosphor layer in the white light-emitting diode package, *J. Lightwave Technol.* 30 (21) (2012) 3376–3380.
- [22] P. Kubelka, New contributions to the optics of intensity light-scattering materials Part I, *J. Opt. Soc. Am.* 38 (5) (1948) 448–457.
- [23] Z.Y. Liu, S. Liu, K. Wang, X.B. Luo, Measurement and numerical studies of optical properties of YAG: Ce phosphor for white LED packaging, *Appl. Opt.* 49 (2010) 247–257.
- [24] W.M. Yen, S. Shionoya, H. Yamamoto, Phosphor Handbook, second ed., Taylor & Francis, New York, 2007. Chap. 4.
- [25] Y.T. Zhu, P. Dutta, N. Narendran, Post-synthesis annealing effects on SrGa<sub>2</sub>Se<sub>4</sub>:Eu<sup>2+</sup> phosphors with peak emission wavelength in the green gap, *Proc. SPIE* 7784 (2010) 77840P.
- [26] C. Sommer, F. Reil, J.R. Krenn, P. Hartmann, P. Pachler, H. Hoschopf, F.P. Wenzl, The impact of light scattering on the radiant flux of phosphor-converted high power white light-emitting diodes, *J. Lightwave Technol.* 29 (15) (2011) 2285–2291.
- [27] N.T. Tran, J.P. You, F.G. Shi, Effect of phosphor particle size on luminous efficacy of phosphor-converted white LED, *J. Lightwave Technol.* 27 (22) (2009) 5145–5150.
- [28] R. Hu, X. Fu, Y. Zou, X.B. Luo, A complementary study to toward scatter-free phosphors in white phosphor-converted light-emitting diodes: comment, *Opt. Express* 21 (4) (2013) 5071–5073.
- [29] H.K. Park, J.H. Oh, Y.R. Do, Toward scatter-free phosphors in white phosphor-converted light-emitting diodes, *Opt. Express* 20 (9) (2012) 10218–10228.