An improved model for predicting thermal contact resistance at liquid–solid interface

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ABSTRACT

Polymeric fluidic thermal interface materials (TIMs) are commonly utilized to reduce the thermal resistance between two solid surfaces in microelectronics. When filling TIMs between the solid surfaces, an amount of air is entrapped between the liquid and solid surfaces, resulting in a thermal contact resistance ($R_c$) arising at the liquid–solid interface. $R_c$ plays an important role on the thin TIMs with high thermal conductivity, while analytical models for $R_c$ at liquid–solid interface are scarce. Based on former researches, this paper developed an improved predictive model through conducting the wettability analysis at the interface. The model expresses $R_c$ as a function of topography of solid surface, wettability of the liquid on rough surface, mechanics of contact at the interface, and thermal parameters of the contacting bodies. $R_c$ at the TIM–aluminium interface was measured for comparison with the model results. The comparison shows that the model results matches to experimental data within 14.3% when the applied pressure is 0.1 MPa, and the deviations are still within the error bars at higher pressure.

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1. Introduction

Thermal interface materials (TIMs) are commonly used in the thermal management of microelectronics. Polymeric fluidic TIMs, such as thermal greases and phase change materials, are often utilized to reduce the thermal contact resistance ($R_c$) between the die and the heat spreader, and between the heat spreader and the heat sink [1–3]. Although the polymeric TIMs with high thermal conductivity lead to low bulk thermal resistance ($R_{bulk}$), thermal resistance at the TIM bond lines is often higher than expected. One of the most important reasons is that when filling the polymeric fluidic materials between the solid surfaces, an amount of air is normally entrapped inside the microcavities of the rough solid surface, resulting in a $R_c$ arising at the liquid–solid interface [1,4]. $R_c$ plays an important role on the thin TIMs with high thermal conductivity [3], while good physical-based analytical models for $R_c$ at liquid–solid surface are still lacking in literatures reviewed carefully in Ref. [3]. It is therefore necessary to develop an effective model to predict it.

Past literatures abound with the models of $R_c$ in solid–solid contact conditions [5–8]. These studies have all concluded that $R_c$ is a function of the surface roughness, asperity slope, mechanical properties of the solid bodies, apparent area of contact and the compressive load between the contacting surfaces. However, these models are not suitable for the polymeric fluidic TIMs because the mechanical properties, such as Young’s modulus or hardness, cannot be defined by the same way [4].

Two approaches have been used to predict $R_c$ according to the limited literatures. One way is to use a numerical model of heat transfer within the solid substrate and the liquid, and to account for temperature gradients within them [9–10]. Bennett and Poulikakos [9] developed a model for estimating thermal contact coefficient between the molten metal and substrate, and they find that the pockets of entrapped air cause indentations on the bottom surface of molten metal. Liu et al. [10] also applied the numerical method to conduct thermal contact analysis for a molten metal drop impacting on a substrate, their results suggest that the type of substrate material effect the thermal contact coefficient a lot. The second way is to follow the method at the solid–solid contact conditions [5–8] to build theoretical models at the liquid–solid interface by conducting topographical and mechanical analyses at the interface [4,11–12]. Prasher [4] developed a surface chemistry model for the fluidic TIMs–solid contacts based on the definition of $R_c$ proposed by Madhusudana [13]. The advantage of such model is taking into consideration the wettability of the liquid on rough surface and making mechanical analysis in detail. Thus the model successfully illustrated the interface characteristics at
Nomenclature

- \(a_i\) radius of microcontacts, m
- \(b_i\) radius of heat flux tube, m
- BLT bond line thickness, m
- \(k\) thermal conductivity, W m\(^{-1}\) K\(^{-1}\)
- \(L\) distance between the marks, m
- \(m\) slope of real surface profile
- \(n_s\) microcontacts density of solid surface, m\(^{-2}\)
- \(P_0\) applied pressure, Pa
- \(P_r\) capillarity pressure of liquid, Pa
- \(P_0\) atmospheric pressure, Pa
- \(q\) heat flux, W m\(^{-2}\)
- \(R_s\) arithmetic average of the surface roughness, m
- \(R_{bulk}\) bulk thermal resistance of TIM, m\(^{2}\) K W\(^{-1}\)
- \(R_c\) thermal contact resistance, m\(^{2}\) K W\(^{-1}\)
- \(R_0\) root mean average of the surface roughness, m
- \(R_{mean}\) mean peak spacing, m
- \(R_{tot}\) total resistance at the TIM bond lines, m\(^{2}\) K W\(^{-1}\)
- \(r_c\) thermal resistance of a single microcontact spot, K W\(^{-1}\)
- \(r^2\) correlation coefficients
- \(T_0\) atmospheric temperature, K
- \(T_c\) contact temperature at the liquid-solid interface, K
- \(V_0, V_c\) volume of the entrapped air before and after the liquid–solid contact, m\(^3\)
- \(Y\) height of the entrapped air after the liquid–solid contact, m

Greek

- \(\sigma\) standard deviation of the asperities heights, m
- \(\phi(y)\) Gaussian distribution function of the heights of asperities
- \(\varepsilon\) factor related to the surface profile
- \(\gamma\) surface energy, N m\(^{-2}\)
- \(\theta\) contact angle, deg
- \(\varphi\) angle between the cavity and mean plan, deg
- \(\Delta T\) temperature difference of the bond lines, K
- \(\Delta R\) error in measurement of thermal resistance, m\(^{2}\) K W\(^{-1}\)
- \(\Delta BLT\) error in measurement of BLT, m

Subscript

- 1, 2 Sides
- \(d\) dispersion component of surface energy
- \(l\) Liquid
- \(p\) polar component of surface energy
- \(s\) Solid

The liquid–solid contact. The surface geometry assumption in this model is that the solid asperities are conical in shape with the same heights, and the spacing of the peaks and valleys is identical and equal to the roughness sampling interval [4]. However roughness sampling interval is a profilometer parameter that is used to modify the roughness measurement conditions [14]. On the other hand, Hamasaiid et al. [11–12] developed a predictive model for the casting-die interface based on the definition of \(R_c\) in CMY model [6]. The advantages of this model are assuming the asperity heights follow a Gaussian distribution which is more acceptable for exhibiting the surface geometry [6], and using mean asperity peak spacing \((R_{mean})\), measured by the profilometer, as the spacing of the peaks and valleys. Different from surface chemistry model, mechanical analysis in Hamasaiid et al. model are completed by evaluating the capillarity pressure of the molten alloy as a function of the applied pressure through experiments. Predictions of the model are found to agree with the experimental results.

According to the above review, Hamasaiid et al. model is found to have better topographical analysis, while wettability analysis in the surface chemistry model makes it characteristic-based and suitable for different types of liquid–solid interfaces. Thus the idea of this paper is to develop an improved model based on Hamasaiid et al. model by adopting the wettability analysis at the liquid–solid interface. This improved model is compared with the experimental results of \(R_c\) at the TIMs–solid interface.

2. Model of thermal contact resistance

2.1. Theoretical background

Fig. 1 schematically shows the formation of \(R_c\) at the fluidic TIM–solid interface. Heat flux passing from the solid to TIM or from TIM to solid needs to be constricted at the microcontact spots. At the macroscopic scale, it brings in a temperature difference at the interface resulting in a thermal resistance. Based on CMY model [6], the thermal field at the interface can be split into several identical isothermal flux tubes as illustrated in Fig. 1. The thermal resistance of a single microcontact spot can be determined from the following relationship [6]:

\[
r_c = \frac{1}{2k_i a_i} \left(1 - \frac{b_i}{a_i}\right)^{1.5}
\]

where \(k_i = 2k_1/k_2/(k_1 + k_2)\) is the effective thermal conductivity of the contacting bodies, \(a_i\) is the radius of the circular microcontact point and \(b_i\) is radius of the heat flux tube. Considering the microcontacts density \(n_i\) of the interface, the overall thermal resistance is the sum of all the single microcontact resistances which are connected in parallel:

\[
R_c = R_{mean} = \frac{1 - \left(\frac{a_i}{b_i}\right)^{1.5}}{2k_i n_i (a_i)}
\]

where \(a_i\) and \(b_i\) are the average radii of the microcontact points and the heat flux tubes, respectively. Eq. (2) has been widely accepted in the field of \(R_c\). Following the ways of developing \(R_c\) model at the solid–TIM interfaces [5–8,15–19], \((a_i), (b_i)\) and \(n_i\) can be estimated by topographical and mechanical analyses at

![Fig. 1. Formation of \(R_c\) and the resistance components in the solid–TIM–solid structure.](image-url)
liquid–solid contacts. In this paper, asperities resistance is neglected because it is small enough compared to \( R_c \) [12].

### 2.2. Topographical analysis

To describe the solid surface profile, three key surface roughness parameters are considered: the arithmetic mean deviation of the profile \( R_a \), root-mean-square deviation of the profile \( R_q \), and \( R_{sm} \). Hamasaiid et al. [12] have made three assumptions to simplify the solid surface profile:

1. The solid surface profile consists of successive peaks and valleys with the bases on the mean plane. The spacing of the peaks and valleys is identical and equal to \( R_{sm} \).
2. The asperities are conical in shape with the same slope (\( m \)).
3. The heights of asperities (\( y \)) follow a Gaussian distribution with a distribution function \( \phi(y) \) which is expressed as:

\[
\phi(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-y^2}{2\sigma^2}\right) \quad y \in (-\infty, +\infty)
\]

Based on the standard definitions, \( R_a \) and \( R_q \) can be expressed as:

\[
R_a = \int_{-\infty}^{+\infty} \phi(y) |y| dy
\]

\[
R_q = \sqrt{\int_{-\infty}^{+\infty} \phi(y)y^2 dy} = \sigma
\]

Eq. (5) shows that \( R_q \) is equal to the standard deviation of the Gaussian distribution \( \sigma \).

For the convenience of mechanical analysis, the model proposed by Hamasaiid et al. [12] modifies the real solid surface profile to a new profile as shown in Fig. 2. The modified surface profile has a common base for the asperities. Then all the valleys are brought to the same level \( y = 0 \). The asperities are still conical in shape but with a new normalized slope \( m_n \). To keep initial surface roughness parameters \( (R_a, R_q, R_{sm}) \) measured by profilometer constant, Hamasaiid et al. [12] propose a new height distribution of peaks \( \phi_n(y) \) for the modified profile which is expressed by equation:

\[
\phi_n(y) = \frac{2}{\sqrt{2\pi}\sigma} \exp\left(\frac{-y^2}{2\sigma^2}\right) \quad y \in [0, +\infty)
\]

After the transformation of solid profile, the liquid–solid contact profile is shown in Fig. 3. The authors [12] have successfully developed the expressions of \( \langle a_n \rangle \), \( \langle b_n \rangle \) and \( n_n \), respectively as follows:

\[
\langle a_n \rangle = \int_{y=0}^\infty (y - Y) \frac{m_n}{m} \phi_n(y)dy
\]

\[
= \frac{R_{sm}}{2} \left( \exp\left(\frac{-Y^2}{2\sigma^2}\right) - \sqrt{\frac{\pi}{2}} \sigma \text{erfc}\left(\frac{Y}{\sqrt{2\sigma}}\right) \right)
\]

(7)

\[
\langle b_n \rangle = \frac{R_{sm}}{2}
\]

(8)

\[
n_n = \frac{8}{\pi\sigma} \left( \frac{1}{R_{sm}} \right)^2 \text{erfc}\left(\frac{Y}{\sqrt{2\sigma}}\right)
\]

(9)

where \( Y \) is the mean height of the trapped air, \( c \) is a factor determined statistically to be close to 1.5 [12] and \( m_n \) is given by:

\[
m_n = 2\sqrt{2} \frac{\sigma}{\pi R_{sm}}
\]

(10)

By substituting Eqs. (7)–(9) to Eq. (2), the thermal contact resistance is determined as follow:

\[
R_c = \frac{8k_c}{1.5\pi R_{sm} \text{erfc}\left(\frac{Y}{\sqrt{2\sigma}}\right) \left( \exp\left(\frac{-Y^2}{2\sigma^2}\right) - \sqrt{\frac{\pi}{2}} \sigma \text{erfc}\left(\frac{Y}{\sqrt{2\sigma}}\right) \right)^{1.5}}
\]

(11)
According to the above equations, $R_c$ is a function of $k_c$, surface roughness ($\sigma$, $R_{nm}$) and $Y$. To determine $Y$, mechanical analysis should be done at the contacting interface.

2.3. Mechanical analysis

Fig. 4 shows the wetting of the liquid on the solid rough surface. When liquid material impinges on the solid surface, it spreads out and wets the microcavity inside which the air is then compressed. An equilibrium state is established at the liquid–solid interface between the applied pressure $P_b$, the capillarity pressure $P_c$ due to the liquid surface energy, and the back pressure of air $P_b$. Assuming the entrapped air is ideal gas and not allowed to escape, the ideal gas equation can express the equilibrium as follow:

$$\frac{(P_b + P_c)V_c}{T_c} = \frac{P_b V_0}{T_0}$$

where $V_0$ and $V_c$ are the volume of the entrapped air before and after the liquid–solid interface, $T_0$ and $T_c$ are the initial and contact temperature at the interface, $+$ or $-$ means good or poor surface wetting, respectively. To solve Eq. (12), $P_c$ must be given first. In Hamasaid et al. model [12], an empirical expression of $P_c$ has been evaluated through experiments at the casting–die interface. In this paper, we try to adopt the wettability analysis of surface chemistry model to solve this problem. In surface chemistry model [4], $P_c$ is found to be determined by the surface tension of liquid, wetting conditions, and the topography of microcavity. For this problem, $P_c$ is given by [4]:

$$P_c = \frac{2\gamma_l \sin(\theta + \varphi)}{Y \cot(\varphi)}$$

where $\gamma_l$ is the surface energy of liquid, $\theta$ is the contact angle of liquid on solid surface and $\varphi$ is the angle between the cavity and mean plan which can be expressed as:

$$\varphi = \arctan(m_n)$$

According to the above topographical analysis and assumption that the entrapped air is conical in shape, the average values of $V_0$ and $V_c$ can be expresses as [12]:

$$V_c = \frac{1}{3} \pi \frac{1}{m_n} \int_{y=0}^{Y} \phi_b(y)y^3 dy \approx \frac{\pi Y^3}{3m_n}$$

$$V_0 = \frac{1}{3} \pi \frac{1}{m_n} \int_{y=0}^{\infty} \phi_b(y)y^3 dy = \frac{2\pi}{3} \sqrt{\frac{2\sigma^3}{\pi m_n}}$$

By substituting Eqs. (13), (15) and (16) to Eq. (12), the equilibrium equation can be expressed as follows:

$$\left(\frac{P_b - P_c \sin(\theta + \varphi)}{Y \cot(\varphi)}\right) \frac{1}{T_c} \frac{V_0}{V_c} = \frac{P_b}{T_0} \frac{2\sqrt{2\sigma^3}}{Y^3}$$

For the contact angle of fluidic TIMs on substrate is less than 90°, which will be illustrated in detail in Section 3.3, this paper deals with the good wetting condition. Thus Eq. (17) can be solved to determine the $Y$ for the microcavities:

$$Y \approx \frac{1}{1 + \frac{\gamma_l \cot(\varphi)}{3P_b Y^3 \phi_b(y) \phi_a(y) \cot(\varphi)}}$$

where

$$\Gamma = \left(\frac{P_b^2}{4P_b \phi_a(y) \phi_b(y) \phi_c(y)} \frac{8\sigma^3}{27P_b \phi_a(y) \phi_c(y) \phi_b(y)^2 \cot(\varphi)}\right)^{1/2} + \frac{P_b}{2P_b \phi_c(y)^2 \phi_a(y)}$$

Eqs. (18)–(20) shows that $Y$ depends on applied pressure ($P_b$), wetting ability ($\gamma_l$, $\theta$), topography of the solid surface ($m_n$, $R_{nm}$) and temperature ($T_c$, $T_0$). These parameters together with $R_{nm}$ and $k_c$ determine the $R_c$ according to Eq. (11). In the following, we will validate the improved model via experiments and discuss the influences of $\sigma$, $R_{nm}$, $k_c$ and $P_b$ on $R_c$ in detail.

3. Experiment for the measurement of $R_c$

3.1. Experimental principles

Experimental measurements are conducted on three layers structure as shown in Fig. 1. The total resistance $R_{tot}$ at the TIM bond lines can be expressed as:

$$R_{tot} = \frac{\Delta T}{q} = R_{bulk} + R_{c1} + R_{c2}$$

where $\Delta T$ is the temperature difference of the bond lines, $q$ is the heat flux through the bond lines, $R_{bulk}$ is equal to the ratio of bond line thickness (BLT) to thermal conductivity of TIM ($k_{TIM}$), and $R_{c1}$ is equal to $R_{c2}$ when the roughness of upper and lower substrates are assumed to be identical. Thus, $R_{tot}$ can be given by:

$$R_{tot} = \frac{1}{k_{TIM}} BLT + 2R_c$$

From Eq. (22), Fig. 5 shows the plots of $R_{tot}$ versus BLT for multiple materials. It can be seen that collecting $R_{tot}$ data over a range of BLT will create a linear relation with a slope $1/k_{TIM}$ and a vertical axis intercept $2R_c$. Thus, based on this principle, both $k_{TIM}$ and $R_c$ can be measured.

3.2. Thermal and BLT measurement

The $R_c$ and $k_{TIM}$ were measured using a thermal interface material testing system which is based on ASTM D 5470 standard [20]. Fig. 6 illustrates the sketch of testing system consisting of five main components: heat flux meters, heating and cooling units, load fixture, insulation materials, and the camera to measure the BLT.
Heat flow through the specimen is measured by the heat flux meters with a 30 mm × 30 mm cross sectional area. It is accomplished by measuring the linear temperature gradient dT/dx in the heat flux meters and using the Fourier’s law of heat conduction $q = k(dT/dx)$. The centerline temperature is measured at 10 mm intervals along each of the flux meter using five 1.5 mm diameter platinum resistance temperature detectors (RTDs).

Power is applied by a heater block embedded with four wire wound cartridge heaters capable of 120 W. A water-cooled heat sink is designed to remove the heat rejected from the upper heat flux meter. The design uses a micro-pump with a maximum flow rate $2.3 \times 10^{-5}$ m$^3$/s to circulate water. Consequently, heat flow in flux meter is assured to be one-dimensional. The contact pressure on the specimen is controlled by the lead screws. A load cell with a resolution 3.3 kPa continuously monitors the applied load.

Insulation tapes with a thermal conductivity of 0.034 W m$^{-1}$ K$^{-1}$ and a thickness of 6 mm is placed around four sides of the heat flux meters to ensure the heat flow through the lower and upper meters varies by less than 5%. 10 mm thick firebond insulation is placed around four sides of heat block to eliminate heat leakage to the environment. Heat loss from the bottom is minimized by attaching it to 10 mm thick FR-4 epoxy material.

A camera with a microscope lens is implemented to measure BLT, as well as to detect the parallelism of the interface before the start of each test. Two 1 mm × 1 mm square marks are attached to the upper and lower flux meters located approximately 2 mm from the flux meters edges. These marks are used as targets and the camera measures the distance between the centers of the targets. These marks are scanned in an approximately 5 mm field of view. The camera provides 1024 × 1024 bit resolution and each pixel is approximately 5 μm in length, so the resolution of BLT measurement is limited to 5 μm. There are two steps to measure BLT. The first step is to apply a 1 MPa load between the heat flux meters without TIM and measure the targets distance $L_0$. The second step is to separate the flux meters, fill the TIM between the flux meters and measure the targets distance $L_1$ at the desired pressure. Then BLT is calculated by subtracting the $L_0$ from $L_1$.

For the measurement of $R_s$ and $k_{TIM}$, BLT of TIMs needs to be controlled so that $R_{tot}$ can be measured for a range of BLT as mentioned above. In this experiment, stainless steel shims are mixed into the TIMs to control the thickness as shown in Fig. 7. The shims are 1.5 mm in diameter, and the thermal conduction through them is negligible which has been verified by Prasher et al. [21]. Although the pressure from lead screws is applied on the sample, the shims take most of the pressure.

### 3.3. Sample preparation

Aluminium-6061 T4 blocks were chosen as the solid substrates, as well as the heat flux meters. The thermal conductivity is 154 W m$^{-1}$ K$^{-1}$. Four couples of blocks were machined with different surface roughness. For each couple, surface roughness parameters are measured by a Mitutoyo SJ-401 surface profilometer for 12 times, and we set the average value as the roughness of the couple. Shin Etsu KF96H silicone oil and Dow Corning TC5121 thermally conductive grease are prepared for the TIMs. Because this experiment can measure the thermal conductivity, the average values of experimental results are set to be their thermal conductivities. Eight experimental combinations of substrates roughness and TIMs are shown in Table 1.

The surface energy of the silicone oil was provided by the supplier, and that of the grease and aluminium substrate is measured with standard two liquid method using water and diiodomethane (CH$_2$I$_2$) based on the ASTM D 7490 standard [22]. This method can measure the polar and dispersion components of the material surface energy, $\gamma_p$ and $\gamma_d$, respectively. The total surface energy of the material is the sum of $\gamma_p$ and $\gamma_d$. The contact angle of silicone oil was directly measured on the smoothest aluminium substrate by sessile drop method [23]. However, because thermal grease has extremely high viscosity, the method cannot be used for grease. The contact angle of grease on the aluminium substrate can be calculated by using the equation [23]:

\[
(1 + \cos \theta)\gamma_l = 2\sqrt{\gamma_{lp}\gamma_{sd}}
\]

where the subscripts 1 and s refer to the liquid and substrate, respectively. Because $\gamma_p$ and $\gamma_d$ of the grease and substrate have been measured, the contact angle can be calculated. Table 2 shows the surface energy of silicone oil, thermal grease, and aluminium block, and the contact angle of silicone oil and thermal grease on the aluminium block.
3.4. Error analysis

According to Eq. (22), the error in \( R_c \) is given as [24]

\[
\frac{\Delta R_c}{R_c} = \sqrt{\left( \frac{\Delta R}{(R_{\text{tot}} - BLT/k_{\text{TIM}})/2} \right)^2 + \frac{1}{k_{\text{TIM}}^2} \left( \frac{\Delta BLT}{(R_{\text{tot}} - BLT/k_{\text{TIM}})/2} \right)^2}
\]

(24)

Following the method in Ref. [25], the errors in \( R \) for the silicone oil and grease are measured with \( 3.8 \times 10^{-6} \text{ m}^2 \text{ K}^{-1} \text{ W}^{-1} \) and \( 1.3 \times 10^{-6} \text{ m}^2 \text{ K}^{-1} \text{ W}^{-1} \), respectively. The error in BLT is 5 \( \mu \text{m} \) which is equal to the pixel size of the camera.

Table 1
Experimental combinations for the measurement of \( R_c \).

<table>
<thead>
<tr>
<th>Materials</th>
<th>Silicone oil ( (k_{\text{TIM 1}}) )</th>
<th>Grease ( (k_{\text{TIM 2}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium 154 W m(^{-1}) K(^{-1})</td>
<td>( \sigma = 0.23 \mu \text{m} \ R_{\text{sm}} = 99.2 \mu \text{m} )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.34 \mu \text{m} \ R_{\text{sm}} = 90.8 \mu \text{m} )</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 0.72 \mu \text{m} \ R_{\text{sm}} = 132.1 \mu \text{m} )</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 1.13 \mu \text{m} \ R_{\text{sm}} = 166.4 \mu \text{m} )</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2
Surface energy of TIMs and aluminium block, and the contact angle of TIMs on the block.

<table>
<thead>
<tr>
<th></th>
<th>Silicone oil</th>
<th>Thermal grease</th>
<th>Aluminium block</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_S ) (mN m(^{-1}))</td>
<td>–</td>
<td>24.2</td>
<td>41.6</td>
</tr>
<tr>
<td>( \gamma_D ) (mN m(^{-1}))</td>
<td>–</td>
<td>14.0</td>
<td>4.2</td>
</tr>
<tr>
<td>( \gamma_I ) (mN m(^{-1}))</td>
<td>21.3</td>
<td>38.2</td>
<td>45.8</td>
</tr>
<tr>
<td>( \theta ) (deg)</td>
<td>22.3</td>
<td>45.0</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 8. (a) Plots of thermal resistance \( R \) versus BLT; (b) Comparison of the model results with experimental data for silicone oil.

4. Validation and discussion

4.1. Comparison of experimental data with the model

For the all experimental combinations, experiments were conducted for four different BLTs to find out \( R_c \) and \( k_{\text{TIM}} \). Figs. 8 and 9(a) show the plots of thermal resistance \( R \) versus BLT for the silicone oil and thermal grease, respectively. It is shown that \( R \) is linearly dependent on BLT, thus \( R_c \) and \( k_{\text{TIM}} \) can be computed by the linear least square method. The solved linear regression equations and correlation coefficients are given in Table 3. According to
Eq. (22), thermal conductivity of silicone oil can be obtained by taking the average of the 1/slope of the groups 1 to 4 linear equations, which is equal to 0.21 W m\(^{-1}\) K\(^{-1}\). In the same way, thermal conductivity of grease can be computed, which is equal to 2.44 W m\(^{-1}\) K\(^{-1}\). Meanwhile, \(R_c\) for each group can be obtained by taking half of the intercept of the linear regression equations. Figs. 8 and 9(b) show the experimental results of \(R_c\) for the silicone oil and thermal grease, respectively, as well as the error bars. According to Eq. (24), the biases of silicone oil and grease from the experiments are 2.41 \(10^{-10}\) m\(^2\) KW\(^{-1}\) and 2.43 \(10^{-6}\) m\(^2\) KW\(^{-1}\), respectively.

As mentioned above, predicting \(R_c\) needs to obtain the parameters as follow: topography parameters (\(\sigma, R_{sm}\)), wettability parameters (\(\gamma_s, \theta\)), mechanical parameter (\(P_b\)), and thermal parameters (\(k_s, T_0, T_c\)). In the experiments of groups 1 to 8, \(P_b\) is 0.1 MPa, \(T_0\) is 288 K, and \(T_c\) is approximately 323 K by taking the average temperature of the TIMs in the testing. Other parameters can be obtained in Tables 1 and 2. Figs. 8 and 9(b) show the model results of \(R_c\) for all groups. Compared with the experimental results, it can be found that the improved model matches well with experimental data. The deviation between the experimental and model results is less than 14.3%.

In order to study the influence of \(P_b\) on \(R_c\), different pressures varied from 0.13 MPa to 0.48 MPa were set on group 7 without the shims controlling the BLT. Fig. 10(a) shows the scatters of
measured $R$ versus measured BLTs. Based on Eq. (22), the values of $R_c$ with various pressures can be computed. Fig. 10(b) shows the comparison between the experimental and model results. At lower pressure, the model matches well with experimental results. But the model predictions tend to deviate at higher pressure owing to the higher relative error in thin BLT measurement.

4.2. Influence of the parameters on the model results

In the Ref. [12], the authors have highlighted the effect of surface roughness on $R_c$ in liquid–solid interface. According to their predictive model, $\sigma$ does not have any influence on $R_c$, while $R_{\text{m}}$ has more effect on $R_c$. Similar results can be found in the improved model. Using the parameters of group 7, Fig. 11 respectively presents the curves of $R_c$ versus $\sigma$ and $R_{\text{m}}$ when one of them keeps constant. It is shown that $R_c$ changes hardly with $\sigma$, but increases with $R_{\text{m}}$ obviously. In order to explain the results, Fig. 12 shows the plots of $\langle a_l \rangle / \langle b_l \rangle$ and $\langle a_l \rangle \times n_e$ as a functions of $\sigma$ and those as a functions of $R_{\text{m}}$. When $R_{\text{m}}$ keeps constant, $\sigma$ nearly has no significant influence on $\langle a_l \rangle / \langle b_l \rangle$ or $\langle a_l \rangle \times n_e$. Thus $R_c$ does not change with $\sigma$ according to Eq. (2). Contrary to $\sigma$, the larger the value of $R_{\text{m}}$, the smaller the $\langle a_l \rangle \times n_e$. And the $\langle a_l \rangle / \langle b_l \rangle$ still remains constant so that $R_c$ turns larger according to Eq. (2). Comparing the $R_c$ between groups 1 and 2 in Fig. 8(b), or between groups 5 and 6 in Fig. 9(b), the experimental results seem to be accorded with the claim that $R_{\text{m}}$ has the more important impact on $R_c$. However, the considerable experimental bias and the limited experimental samples fail to make it credible enough to verify the point. Thus it is desired to investigate this work in the future to validate the results of the improved $R_c$ model.

Comparing the predictive results of $R_c$ between the groups having the same roughness parameters, such as groups 1 and 5, groups 2 and 6, and so on, it can be found that $R_c$ is approximately inversely proportional to thermal conductivity of TIMs. Thermal conductivity of aluminium is much larger than $k_{\text{m}}$ and this makes $k_l$ approximately equal to $2k_{\text{m}}$. Then Eq. (2) shows $R_c$ varies in an inverse proportion to $k_{\text{m}}$.

As illustrated in Fig. 10(b), $R_c$ decreases with $P_b$. To better understand the effect of $P_b$ on $R_c$, $Y$ is plotted as a function of $P_b$ and it is shown in Fig. 13. It shows that $Y$ decreases with $P_b$. Thus higher $P_b$ makes the liquid and solid contact better and result in a lower $R_c$ at the interface.

Finally, the effect of wettability parameters ($\gamma_l$) on $R_c$ is investigated. Figs. 14 and 15(a) respectively presents the curves of $R_c$ versus $\gamma_l$ and $\theta$ for the groups 5–8. From Fig. 14(a), it is seen that $R_c$ tendentially decreases with $\gamma_l$ increasing when $\theta$ is equal to $45.0^\circ$. Fig. 14(b) plots $P_l/P_b$ as a function of $\gamma_l$. The figure shows that $P_l$ increases with $\gamma_l$ increasing. Then $Y$ decreases according to Eq. (18) and finally $R_c$ decreases. From the figure, it can be also found that the larger $R_{\text{m}}$ is, the lower $P_l$ is. On the other hand, Fig. 15(a) shows a very slow decrease of $R_c$ with $\theta$ increasing when $\gamma_l$ is relatively small and equal to $38.2 \text{ mN m}^{-1}$. In order to explain the results, $P_l/P_b$ is plotted as a function of $\theta$ as shown in Fig. 15(b). It shows that $P_l$ increase with $\theta$ increasing. However the value of $P_l$ is much smaller than $P_b$ which means $P_l$ makes less contribution to $Y$. So for the materials with small $\gamma_l$, $R_c$ hardly changes with $\theta$.

5. Conclusion

This paper proposes an improved model for predicting thermal contact resistance $R_c$ at the liquid–solid interface based on Hamasaki et al. model. Through conducting the wettability analysis on the liquid–solid contact, an explicit expression is derived to predict the height of entrapped air between the liquid and solid. Experimental measurements have been conducted on TIMs-aluminium...
interface. The results show the model matches well with the experimental data.

The proposed model shows that compared to standard deviation of the asperities heights \( \sigma \), the mean asperity peak spacing \( R_{am} \) has more effect on \( R_c \) and \( R_s \) decreases with \( R_{am} \). It is observed that \( R_c \) varies with thermal conductivity of TIMs \( k_{TM} \) in an inverse proportion, when the thermal conductivity of solid substrate is much larger than \( k_{TM} \). The extrapolation of the height of the entrapped air \( Y \) has shown that it is directly dependent on applied pressure \( P_b \). Higher \( P_b \) can make liquid and solid contact better and results in a lower \( R_c \). \( R_c \) tendentially decreases with surface energy \( \gamma_s \). For the materials with small \( \gamma_s \), \( R_s \) hardly changes with contact angle of liquid on solid surface \( \theta \).

**Conflict of interest**

None declared.

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**References**


