# A Model for Calculating the Bidirectional Scattering Properties of Phosphor Layer in White Light-Emitting Diodes

Run Hu and Xiaobing Luo, *Member, IEEE*

***Abstract*—In this study, a model for calculating the bidirectional scattering properties of phosphor layer was established by considering the light absorption and light conversion of phosphor particle simultaneously. Based on the present model, the light extraction efficiency was calculated and validated by comparing with the existing experimental data and models. It is demonstrated that the present model predicts better. The transmission and reflection intensities of blue light and yellow light were found to be the functions of phosphor thickness. There exists a close critical phosphor thickness for reflected blue light and transmitted yellow light. The critical phosphor thickness is relative to the phosphor concentration.**

***Index Terms*—Bidirectional scattering, light-emitting diodes (LEDs), phosphor conversion.**

## I. Introduction

WHITE light sources based on light-emitting diodes (LEDs) have been extensively used for their extraordinary characteristics over traditional light sources [1]–[4]. Among the synthesizing mechanisms of white light, the phosphor-converting scheme is the most general way, which involves a short-wavelength emitter and long-wavelength downconversion phosphors [5]. In the phosphor-converted LEDs (pc-LEDs), the phosphor particles are dispersed in silicone matrix and then directly or indirectly coated on the LED chip. A fraction of blue light from LED chip is absorbed by the phosphor layer and the rest fraction can be transmitted through the layer by the scattering of phosphor particles. Due to the nonradiative relaxation process in the $4f$- or $5d$- energy level of the doping ions in phosphors, the absorbed blue photon is converted to lower energy yellow photon [6]–[9]. The transmitted blue light and yellow light generates white light by color mixing.

Reobserving the phosphor silicone matrix in microperspective, the light is strongly scattered. Every time the blue/yellow light is scattered by a phosphor particle, light reabsorption and light conversion would happen consequently [10]. The phosphor parameters, such as particle size, distribution, thickness, and concentration, influence the light scattering, absorption, and conversion processes, and finally determine the optical performance of phosphor layer [11]–[15]. Because of the complicated light conversion processes, most researchers prefer to study the optical properties by measurements. In the measurements, however, the blue light and yellow light are difficult to distinguish for quantification because they are mixed with each other. For modeling these processes, due to the anisotropy of the particle distribution, it is better to take account of the bidirectional scattering properties of phosphor layer, which include the forward scattering component and the backscattering component [16]. However, few histories about modeling the bidirectional scattering properties of phosphor can be found in the literature.

Kang *et al.* developed a 1-D model to describe the light propagating in the phosphor layer in terms of light absorption and conversion, but they neglected the light scattering effect in their model [17]. Li *et al.* proposed a double-layer phosphor package by developing the 1-D model proposed by Kang *et al.*, but they also neglected the light scattering effect [18]. Zhu and Narendran analyzed the performance of multiphosphor configuration in the remote "scattered photon extraction" white LEDs [19]. Ishida *et al.* developed a model to calculate the light output of LEDs with consideration of light scattering based on the Kubelka–Munk method [20]. The authors established the equations from the view of energy levels for each phosphor species.

In this paper, we developed a model to calculate the bidirectional scattering properties of phosphor layer with consideration of light reabsorption and light conversion simultaneously. The light extraction efficiency (LEE) was calculated and compared with existing experimental data and models. The transmission and reflection intensities of blue light and yellow light were investigated by changing the phosphor thickness. The variation trends were analyzed in detail.

## II. Model Setup

As shown in Fig. 1, for a very thin layer with a thickness of $dz$, the forward scattering function and backscattering function are denoted as $I(z)$ and $J(z)$, respectively. Subscripts $B$ and $Y$ stand for blue light and yellow light, respectively. By employing the energy balance, we obtain the following equations.

For the blue light

$$I_B(z+dz) - I_B(z) = -a_B I_B(z) d\xi - s_B I_B(z) d\xi + s_B J_B(z+dz) d\xi \quad (1)$$

Manuscript received June 29, 2012; revised August 10, 2012; accepted September 10, 2012. Date of publication September 12, 2012; date of current version October 17, 2012. This work was supported in part by 973 Project of the Ministry of Science and Technology of China under Grant 2011CB013105, and by National 863 project of the Ministry of Science and Technology of China under Grant 2011AA03A109.

The authors are with the School of Energy and Power Engineering, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: luoxb@mail.hust.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JLT.2012.2218631

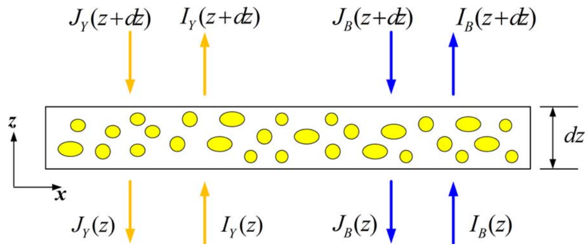


Fig. 1. Schematic of light forward scattering and backscattering in a phosphor layer.

$$J_B(z) - J_B(z+dz) = -a_B J_B(z+dz)d\xi - s_B J_B(z+dz)d\xi + s_B I_B(z)d\xi. \quad (2)$$

For the yellow light

$$I_Y(z+dz) - I_Y(z) = -a_Y I_Y(z)d\xi - s_Y I_Y(z)d\xi + s_Y J_Y(z+dz)d\xi + \frac{1}{2}\eta[I_B(z) + J_B(z+dz)]d\xi \quad (3)$$

$$J_Y(z) - J_Y(z+dz) = -a_Y J_Y(z+dz)d\xi - s_Y J_Y(z+dz)d\xi + s_Y I_Y(z)d\xi + \frac{1}{2}\eta[I_B(z) + J_B(z+dz)]d\xi \quad (4)$$

where $a_B$ and $a_Y$ are the absorption coefficients of blue light and yellow light, respectively. The $s_B$ and $s_Y$ are the scattering coefficients of blue light and yellow light, respectively. $\eta$ is the energy conversion coefficient from blue light to yellow emission. $d\xi$ is the mean path of light path inside the phosphor layer. When the incident light is diffuse and scattering inside the particle layer takes place in all directions, the length of the light path must be longer than the thickness of phosphor layer [21]. The mean light path $d\xi$ could be expressed as [21]

$$d\xi = 2dz. \quad (5)$$

When $dz$ vanishes, (1)–(4) could be rewritten as follows with the substitution of (5):

$$\frac{dI_B(z)}{dz} = -2(a_B + s_B)I_B(z) + 2s_B J_B(z) \quad (6)$$

$$\frac{dJ_B(z)}{dz} = 2(a_B + s_B)J_B(z) - 2s_B I_B(z) \quad (7)$$

$$\frac{dI_Y(z)}{dz} = -2(a_Y + s_Y)I_Y(z) + 2s_Y J_Y(z) + \eta[I_B(z) + J_B(z)] \quad (8)$$

$$\frac{dJ_Y(z)}{dz} = 2(a_Y + s_Y)J_Y(z) - 2s_Y I_Y(z) - \eta[I_B(z) + J_B(z)]. \quad (9)$$

For the phosphor layer coated on LED chips, the plane $z = 0$ means the interface between the LED chip and the phosphor layer. Therefore, the backscattered blue and yellow light projected at the interface will be reflected back to the phosphor layer. The boundary conditions for blue light and yellow light are

$$I_B(0) = I_0 + \gamma_B J_B(0), J_B(d) = 0 \quad (10)$$

$$I_Y(0) = \gamma_Y J_Y(0), J_Y(d) = 0 \quad (11)$$

where $I_0$ is the intensity of incident blue light from LED chip and $d$ is the thickness of the phosphor layer. $\gamma_B$ and $\gamma_Y$ are the reflection coefficients of blue light and yellow light, respectively.

The general solutions to (6) and (7) are

$$I_B(z) = A(1-\beta)e^{\alpha z} + B(1+\beta)e^{-\alpha z}$$
$$J_B(z) = A(1+\beta)e^{\alpha z} + B(1-\beta)e^{-\alpha z} \quad (12)$$

where $A$ and $B$ are the undetermined constant coefficients and

$$\alpha = 2\sqrt{a_B(a_B + 2s_B)}$$
$$\beta = \sqrt{a_B/(a_B + 2s_B)}. \quad (13)$$

With the boundary conditions in (10), the solutions for the blue light in (6) and (7) can be obtained. The coefficients $A$ and $B$ in (12) are expressed as

$$A = I_0 \frac{(\beta - 1)(1+\rho_B)e^{-\alpha d}}{2(1+\beta)(\rho_B + \beta)e^{\alpha d} - 2(1-\beta)(\rho_B - \beta)e^{-\alpha d}}$$
$$B = I_0 \frac{(1+\beta)(1+\rho_B)e^{\alpha d}}{2(1+\beta)(\rho_B + \beta)e^{\alpha d} - 2(1-\beta)(\rho_B - \beta)e^{-\alpha d}} \quad (14)$$

where $\rho_B = (1-\gamma_B)/(1+\gamma_B)$. By substituting the particular solutions for the blue light to (8) and (9), we obtain the general solutions for yellow light as

$$I_Y(z) = C(1-\nu)e^{\mu z} + D(1+\nu)e^{-\mu z} + \frac{2\eta}{\nu(\mu^2 - \alpha^2)}[A(\mu - \nu\alpha)e^{\alpha z} + B(\mu + \nu\alpha)e^{-\alpha z}]$$
$$J_Y(z) = C(1+\nu)e^{\mu z} + D(1-\nu)e^{-\mu z} + \frac{2\eta}{\nu(\mu^2 - \alpha^2)}[A(\mu + \nu\alpha)e^{\alpha z} + B(\mu - \nu\alpha)e^{-\alpha z}] \quad (15)$$

where $C$ and $D$ are the undetermined constant coefficients and

$$\mu = 2\sqrt{a_Y(a_Y + 2s_Y)}$$
$$\nu = \sqrt{a_Y/(a_Y + 2s_Y)}. \quad (16)$$

With the boundary conditions in (11), we obtain the constant coefficients $C$ and $D$ and the particular solutions for the yellow light with the similar method.

## III. Model Validation

When the thickness of the phosphor layer is $d$, the intensities of the transmitted blue and yellow light are the forward scattering intensities at the exit surface $z = d$, i.e., $I_B(d)$ and $I_Y(d)$. Thus, $I_B(d)$ and $I_Y(d)$ determine the LEE $\eta_{\text{LEE}}$, which is normalized as [17], [22], [23]

$$\eta_{\text{LEE}} = (I_B(d) + I_Y(d))/I_0. \quad (17)$$

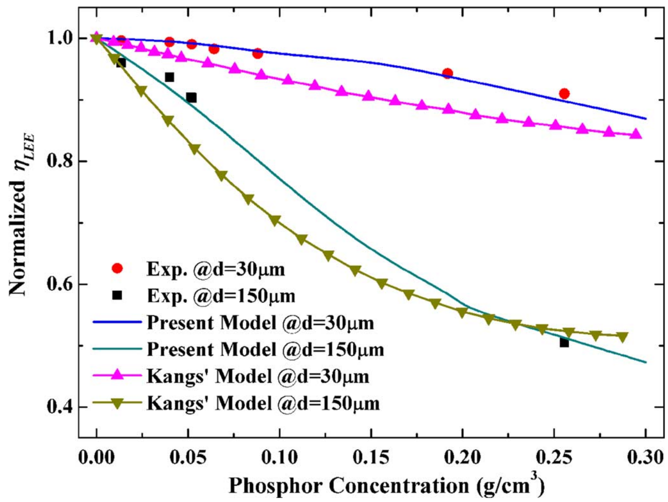


Fig. 2. Variation of normalized LEE with phosphor concentration. For comparison, the experimental results and Kang's model were also plotted [17]. $d$ is denoted as the thickness of the phosphor layer.

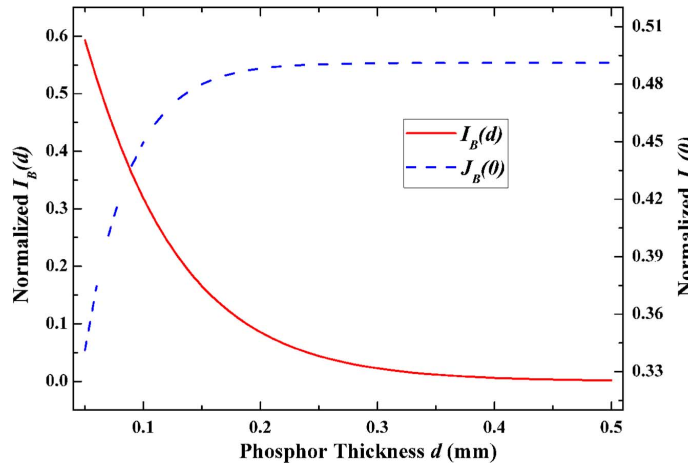


Fig. 3. Variation of normalized transmitted and reflected blue light with phosphor thickness.

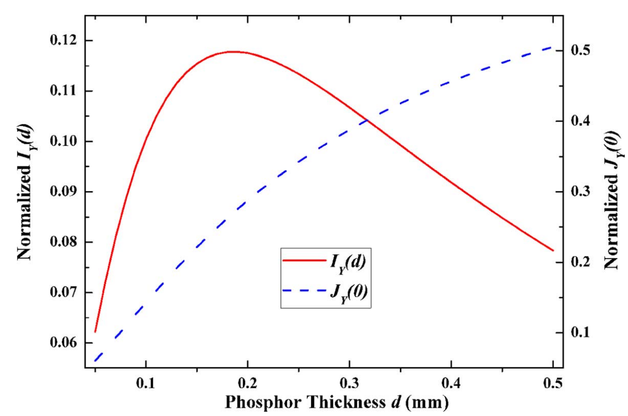


Fig. 4. Variation of normalized transmitted and reflected yellow light with phosphor thickness.

To calculate the LEE, we have to quantify the necessary parameters of the phosphor layer first. The detailed parameters, especially the absorption coefficients and scattering coefficients, were obtained from Mie–Lorenz theory [11]–[13]. The wavelength-dependent absorption coefficients and scattering coefficients, which varied with the changes of phosphor concentration, can be referred to our previous work [4], [11]–[13], [22], [23]. Other parameters can be referred to Kang *et al.* [17]. The calculation results of normalized $\eta_{\rm LEE}$ with the changes of phosphor concentration were plotted in Fig. 2. Along with our calculations, the experimental results of Kang *et al.* and their model predictions were also plotted for comparison. It is seen that for both phosphor thicknesses, with the increase of phosphor concentration, $\eta_{\rm LEE}$ obtained by Kang's experiments and model, and the present model all have similar decreasing trends. But when compared to Kang's model, the present model is closer to the experimental data.

## IV. Transmitted and Reflected Light

For a phosphor layer, the transmission and reflection intensities of blue light and yellow light are most concerned. When the thickness of the phosphor layer is $d$, the transmission intensities of blue light and yellow light are $I_B(d)$ and $I_Y(d)$, respectively; the reflection intensities of blue light and yellow light are $J_B(0)$ and $J_Y(0)$, respectively. To observe the interactions among the light scattering, reabsorption, and conversion inside phosphor layer, the variations of $I_B(d)$, $J_B(0)$, $I_Y(d)$, and $J_Y(0)$ with the change of phosphor thickness $d$ are shown in Figs. 3 and 4. The phosphor concentration was chosen as 0.35 g/cm$^3$. In Fig. 3, it can be seen that with the increase of $d$ from 0.05 to 0.5 mm, $I_B(d)$ decreases greatly. $J_B(0)$ has a rapid rise when $d$ is less than 0.2 mm and tends to be flat gradually afterward. In Fig. 4, the increase of $d$ leads to a trend of gradual increase of $J_Y(0)$, but $I_Y(d)$ witnesses a rapid rise at first and a trend of decrease in the end.

These phenomena may be caused due to following cases: 1) when $d$ rises, more blue light is absorbed and scattered, it becomes harder for blue light to penetrate the phosphor layer, thus $I_B(d)$ decreases extraordinarily, which implies a continuous reduction of transmitted blue light. 2) With the increase of $d$, more blue light is backscattered, thus the reflected blue light $J_B(0)$ increases. Since the blue light is backscattered into all the directions, the further increase of $d$ does not contribute to the enhancement of $J_B(0)$ at the incident surface, and $J_B(0)$ tends to be a stable value. Therefore, there exists a critical phosphor thickness for the reflected blue light. It is predicted that the critical phosphor thickness is dependent on the phosphor concentration, geometry, and particle size because the necessary absorption coefficients and scattering coefficients are determined by these phosphor parameters. 3) When $d$ rises, more yellow light is converted, thus $I_Y(d)$ rises at first. When $d$ increases further, the effects of yellow light absorption and backscattering overweigh the yellow light conversion, thus $I_Y(d)$ drops after a critical phosphor thickness. By comparing Figs. 3 and 4, we can find that the drop of $I_B(d)$ is much larger than the increase of $I_Y(d)$; therefore, $\eta_{\rm LEE}$ decreases when the thickness increases from 30 to 150 $\mu$m as shown in Fig. 2. A close critical phosphor thickness value from the comparison between the curves of $I_Y(d)$ and $J_B(0)$ in Figs. 3 and 4 can also be found. 4) With the increase of $d$, more yellow light is converted and backscattered, thus $J_Y(0)$ witnesses a trend of increase, which indicates a continuous enhancement of yellow reflected light.

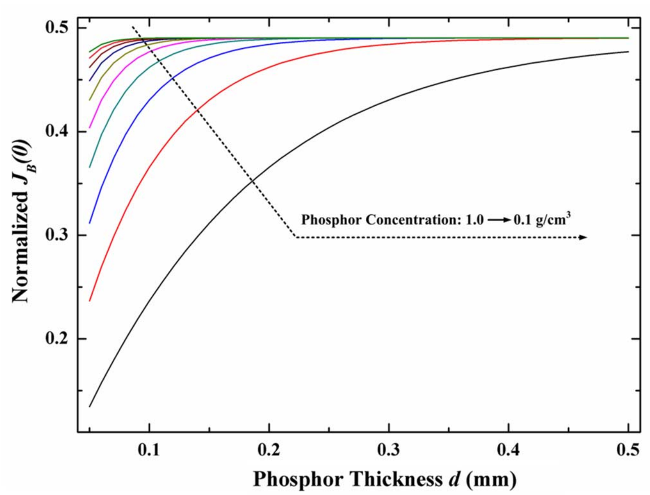


Fig. 5. Variation of normalized reflected blue light with phosphor thickness and concentration.

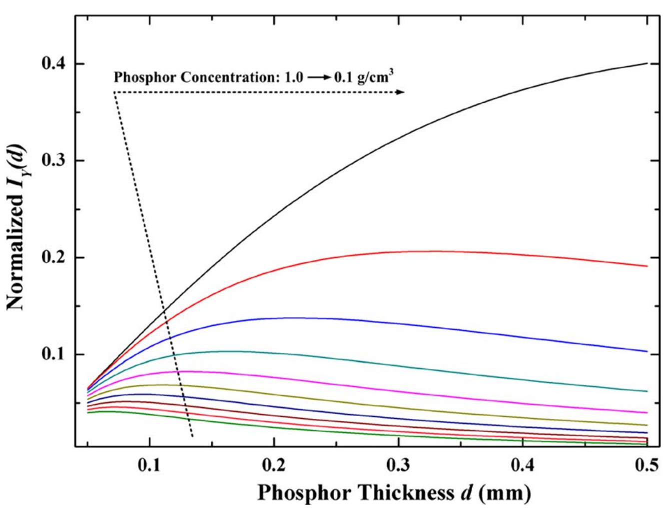


Fig. 6. Variation of normalized transmitted yellow light with phosphor thickness and concentration.

To examine the critical phosphor thickness, the effect of phosphor thickness and concentration on the reflected blue light $J_B(0)$ and transmitted yellow light $I_Y(d)$ was studied. The calculation results are shown in Figs. 5 and 6. It is seen that the phosphor concentration plays an important role in affecting $J_B(0)$ and $I_Y(d)$. The increase of phosphor concentration leads to the decrease of critical phosphor thicknesses for both $J_B(0)$ and $I_Y(d)$. In fact, the increase of phosphor concentration means more phosphor particles in unit volume of phosphor layer, so the light scattering is enhanced. For larger phosphor concentration, $J_B(0)$ and $I_Y(d)$ are easier to reach their peak values; therefore, the critical phosphor thickness is thinned. By comparing Figs. 5 with 6, we can see that the two critical phosphor thicknesses are close to each other for different phosphor concentrations and the reason lies in the interactions of blue light and yellow light, as governed by (8) and (9).

## V. Conclusion

In this study, a model for calculating the bidirectional scattering properties of phosphor layer was established by taking into account of light absorption and light conversion simultaneously. The LEE was calculated and validated with changing phosphor concentration and thickness. It was demonstrated that the present model can predict better than Kang's model. Based on the present model, the transmission and reflection intensities of blue light and yellow light were found to be the functions of phosphor thickness, respectively. There exists a close critical phosphor thickness for the reflection blue light and transmission yellow light. The increase of phosphor concentration contributes to the decrease of the critical phosphor thicknesses for both reflected blue light and transmitted yellow light.

## References


[1] B. L. Wu, X. B. Luo, H. Zheng, and S. Liu, "Effect of gold wire bonding process on angular correlated color temperature uniformity of white light-emitting diode," *Opt. Exp.*, vol. 19, no. 24, pp. 24115–24121, 2011.

[2] N. T. Tran, J. P. You, and F. G. Shi, "Effect of phosphor particle size on luminous efficacy of phosphor-converted white LED," *J. Lightw. Technol.*, vol. 27, no. 22, pp. 5145–5150, Nov. 2009.

[3] H. Zheng, X. B. Luo, R. Hu, B. Cao, X. Fu, Y. M. Wang, and S. Liu, "Conformal phosphor coating using capillary microchannel for controlling color deviation of phosphor-converted white light-emitting diodes," *Opt. Exp.*, vol. 20, no. 5, pp. 5092–5098, 2012.

[4] R. Hu, X. Luo, H. Zheng, Q. Zong, Z. Gan, B. Wu, and S. Liu, "Design of a novel freeform lens for LED uniform illumination and conformal phosphor coating," *Opt. Exp.*, vol. 20, no. 13, pp. 13727–13737, 2012.

[5] D. Bera, S. Maslov, J. W. Lee, J. G. Xue, F. So, and P. H. Holloway, "Optimization of down-conversion phosphor films for high efficiency white organic light-emitting diodes," *J. Photon. Energy*, vol. 1, pp. 011025-1–011025-11, 2011.

[6] J. Andriessen, E. van der Kolk, and P. Dorenbos, "Lattice relaxation study of the 4f-5d excitation of $Ce^{3+}$-doped $LaCl_3$, $LaBr_3$, and $NaLaF_4$: Stokes shift by pseudo Jahn-Teller effect," *Phys. Rev. B*, vol. 76, pp. 075124-1–075124-9, 2007.

[7] M. G. Brik and N. M. Avram, "Electron-vibrational interaction in the 5d states of $Ce^{3+}$ ions in halosulphate phosphors," *Mater. Chem. Phys.*, vol. 128, pp. 326–330, 2011.

[8] A. K. Singh, S. Singh, D. Kumar, D. K. Rai, S. B. Rai, and K. Kumar, "Light-into-heat conversion in $La_2O_3$:$Er^{3+}$-$Yb^{3+}$ phosphor: An incandescent emission," *Opt. Lett.*, vol. 37, no. 5, pp. 776–778, 2012.

[9] Z. G. Ju, S. T. Tan, Z. H. Zhang, Y. Ji, Z. Kyaw, Y. Dikme, X. W. Sun, and H. V. Demir, "On the origin of the redshift in the emission wavelength of InGaN/GaN blue light emitting diodes grown with a higher temperature interlayer," *Appl. Phys. Lett.*, vol. 100, pp. 123503-1–123503-4, 2012.

[10] C. Sommer, F. Reil, J. R. Krenn, P. Hartmann, P. Pachler, H. Hoschopf, and F. P. Wenzl, "The impact of light scattering on the radiant flux of phosphor-converted high power white light-emitting diodes," *J. Lightw. Technol.*, vol. 29, no. 15, pp. 2285–2291, Aug. 2011.

[11] Z. Y. Liu, S. Liu, K. Wang, and X. B. Luo, "Measurement and numerical studies of optical properties of YAG:Ce phosphor for white LED packaging," *Appl. Opt.*, vol. 49, pp. 247–257, 2010.

[12] R. Hu, X. Luo, H. Zheng, and S. Liu, "Optical constants study of YAG:Ce phosphor layer blended with $SiO_2$ particles by Mie theory for white light-emitting diode package," *Front. Optoelectron.*, vol. 5, no. 2, pp. 138–146, 2012.

[13] Z. Y. Liu, S. Liu, K. Wang, and X. B. Luo, "Optical analysis of color distribution in white LEDs with various packaging methods," *IEEE Photon. Technol. Lett.*, vol. 20, no. 24, pp. 2027–2029, Dec. 2008.

[14] C. Sommer, F. Reil, J. R. Krenn, P. Hartmann, P. Pachler, S. Tasch, and F. P. Wenzl, "The impact of inhomogeneities in the phosphor distribution on the device performance of phosphor-converted high-power white LED light sources," *J. Lightw. Technol.*, vol. 28, no. 22, pp. 3226–3232, Nov. 2010.

[15] N. T. Tran and F. G. Shi, "Studies of phosphor concentration and thickness for phosphor-based white light-emitting diodes," *J. Lightw. Technol.*, vol. 26, no. 21, pp. 3556–3559, Nov. 2008.

[16] C. H. Hung and C. H. Tien, "Phosphor-converted LED modeling by bidirectional photometric data," *Opt. Exp.*, vol. 18, no. S3, pp. A261–A271, 2010.

[17] D. Y. Kang, E. B. Wu, and D. M. Wang, "Modeling white light-emitting diodes with phosphor layers," *Appl. Phys. Lett.*, vol. 89, pp. 231102-1–231102-3, 2006.

[18] B. C. Li, D. W. Zhang, Y. S. Huang, Z. J. Ni, and S. L. Zhuang, "A new structure of multi-layer phosphor package of white LED with high efficiency," *Chin. Opt. Lett.*, vol. 8, no. 2, pp. 221–223, 2010.

[19] Y. T. Zhu and N. Narendran, "Investigation of remote-phosphor white light-emitting diodes with multi-phosphor layers," *Jpn. J. Appl. Phys.*, vol. 49, pp. 100203-1–100203-3, 2010.

[20] K. Ishida, I. Mitsuishi, Y. Hattori, and S. Nunoue, "A revised Kubelka–Munk theory for spectral simulation of phosphor-based white light-emitting diodes," *Appl. Phys. Lett.*, vol. 93, pp. 241910-1–241910-3, 2008.

[21] P. Kubelka, "New contributions to the optics of intensity light-scattering materials—Part I," *J. Opt. Soc. Amer.*, vol. 38, no. 5, pp. 448–457, 1948.

[22] R. Hu, X. B. Luo, and S. Liu, "Study on the optical properties of conformal coating light-emitting diode by Monte Carlo simulation," *IEEE Photon. Technol. Lett.*, vol. 23, no. 22, pp. 1673–1675, Nov. 2011.

[23] R. Hu, X. B. Luo, H. Feng, and S. Liu, "Effect of phosphor settling on the optical performance of phosphor-converted white light-emitting diodes," *J. Lumin.*, vol. 132, pp. 1252–1256, 2012.

**Run Hu** received the B.E. degree in energy and power engineering from the Huazhong University of Science and Technology, Wuhan, China, in 2010, where he is currently working toward the Ph.D. degree in engineering thermophysics.

His current research interests include optical and thermal analysis of light-emitting diodes packaging.

**Xiaobing Luo** (M'07) received the B.E. degree in thermal energy and powering engineering and the M.E. degree in engineering thermophysics from the Huazhong University of Science and Technology, Wuhan, China, in 1995 and 1998, respectively, and the Ph.D. degree in engineering thermophysics from Tsinghua University, Beijing, China, in 2002.

From 2002 to 2005, he was with Samsung Electronics, Seoul, Korea, as a Senior Engineer. In 2005, he became an Associate Professor with the School of Energy and Power Engineering, Huazhong University of Science and Technology, where in 2007, he became a Full Professor. He is also a Professor with the Wuhan National Laboratory for Optoelectronics, Wuhan. He has published more than 60 journal papers and granted 20 patents in U.S. and China. His current research interests include light-emitting diode, heat and mass transfer, microfluidics, microelectromechanical systems, sensors, and actuators.