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Directional heat transport through thermal reflection meta-device

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Directional heat transfer may be hard to realize due to the fact that heat transfer is diffusive. In this paper, we try to take one step forward based on the transformation thermodynamics. A special structure and meta-device is proposed to "reflect" the heat flow directionally–just like the mirror to the light beam, in which the heat flow just one-time changes the direction rather than gradually changing the directions in isotropic materials. The benefits of such thermal reflection meta-device are discussed by comparing the corresponding thermal resistance with the same structures of isotropic materials. The proposed meta-device is verified to possess the low thermal resistance and high heat transfer ability with least energy loss, and can be made by nature-existing isotropic materials with specific structures. [http://dx.doi.org/10.1063/1.4973309]

I. INTRODUCTION

The intrinsic diffusive characteristic distinguishes heat from acoustic and electromagnetic waves, which erects barriers for the manipulating of heat transport and results in much energy loss in heat transfer process.[1–5] According to the Fourier's law of heat conduction, the heat flux in the ith direction (q_i) under a given temperature gradient in the jth direction (∇T_j) can be expressed as $q_i = -\kappa_{ij}\nabla T_j$, where κ_{ij} ($= \begin{bmatrix} \kappa_{xx} & \kappa_{xy} \\ \kappa_{yx} & \kappa_{yy} \end{bmatrix}$) is the second-order thermal conductivity tensor of the materials. It is seen that the direction of heat flow is just the opposite direction of temperature gradient. Along the heat flow, heat will obey the thermal extremum principle whereby the propagation of heat takes the path of least thermal resistance,[6] thus more heat tends to flow to the place with higher local thermal conductivity. Due to the diffusive characteristic, the directional transport of heat energy seems like a dream though researchers still have been thinking about that and explored early in theory.[7] Recently, the thermal metamaterials with anisotropic material parameters or an engineered sub-structure of natural materials have gained much popularity.[1,8–10] With thermal metamaterials, many new heat transfer phenomena have been reported, like cloaking, concentrating, rotating, inversing, thermal diode, camouflage, illusion, lens, etc.[11–19] An intuitive question thus arises that whether we could realize the directional heat transport through thermal metamaterials.

In the paper, we try to answer the question by trying to design a kind of thermal reflection meta-devices in order to tune the heat flow to a certain direction. The benefits of such materials are also discussed in terms of thermal resistance and compare with the same structures of isotropic materials.

II. DESIGN METHOD

Fig. 1 shows the primary goal of our design with analog to the light reflection by a mirror. Assuming that the incident light beam is parallel to the x-axis direction and the outgoing light beam is along the y-axis direction, we can easily put a flat mirror in the dash-line box to solve the problem.

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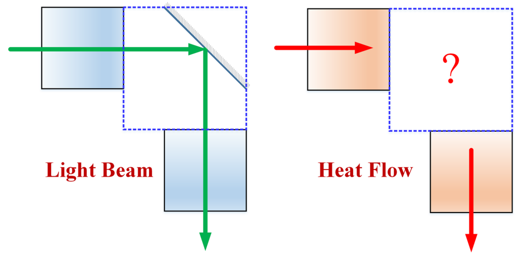

FIG. 1. Schematic of light reflection and thermal flow problem.

The incident and outgoing beams obey the Snell law. When the mirror is placed by 135° with respect to x-axis, the direction of the incident light beam can be change by 90° easily. But can we change the direction of heat flow by 90° similarly? If so, what kind of materials and structures are needed to put in the same dash-line box?

To design such structures, we use a Descartes coordinate transformation.[20,21] As shown in Fig. 2, we map the triangles BCD and ABD in virtual space (*x, y, z*) into BPQ and ABQ in real space (*x', y', z'*), respectively. Note that the coordinates of points A, B, C, D, P, Q are constant and can be expressed as (x_A, y_A), (x_B, y_B), (x_C, y_C), (x_D, y_D), (x_P, y_P), and (x_Q, y_Q), respectively. The coordinate transformation equations from triangle BCD to triangle BPQ can be expressed as

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{1}$$

where $[a_1, b_1, c_1]^T = \boldsymbol{M}^{-1}[x_B, x_P, x_Q]^T$, $[a_2, b_2, c_2]^T = \boldsymbol{M}^{-1}[y_B, y_P, y_Q]^T$, and $\boldsymbol{M} = [x_B, y_B, 1; x_C, y_C, 1; x_D, y_D, 1]$. Based on principles of transformation thermodynamics,[22,23] the thermal conductivity of the triangle BPQ in the real space can be obtained as

$$\kappa_{ij} = \begin{bmatrix} \kappa_{xx} & \kappa_{xy} \\ \kappa_{yx} & \kappa_{yy} \end{bmatrix} = \frac{J\kappa_0 J^T}{\det(J)} \tag{2}$$

where κ_0 is the thermal conductivity of the triangle BCD in the virtual space, and J is the Jacobian matrix of the coordinate transformation, as

$$J = \begin{bmatrix} \dfrac{\partial x'}{\partial x} & \dfrac{\partial x'}{\partial y} \\ \dfrac{\partial y'}{\partial x} & \dfrac{\partial y'}{\partial y} \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}. \tag{3}$$

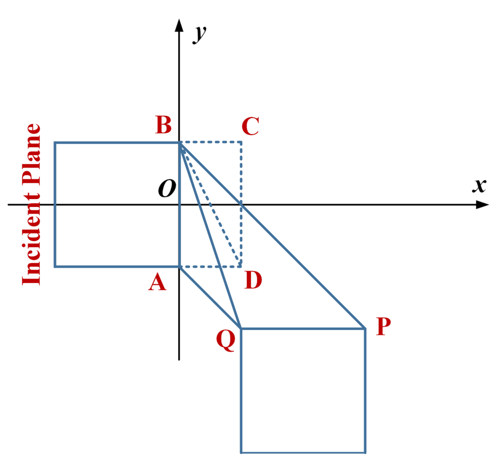

FIG. 2. Schematic illustration of coordinate transformation.

Similarly, we can also map the triangle ABD into the triangle ABQ and obtain the thermal conductivity of ABQ.

III. RESULTS AND DISCUSSIONS

To verify our design and the original idea, we employ the COMSOL package to simulate the temperature profile of the heat conduction by finite-element method. First, we set the coordinates of points A, B, C, D as (0, −1), (0, 1), (1, 1), (1, −1), respectively. In order to turn the direction of heat flow by 90°, the coordinates of points P and Q are set as (3, −2) and (1, −2), respectively. The incident and exit planes are kept at 400 K and 300 K, respectively. The thermal conductivity of the triangles BPQ and ABQ are calculated from Eq. (2). Except for these two triangles, other materials are isotropic natural materials with constant thermal conductivity of κ_0 =1 W/(m·K). To validate the advantages of the proposed design, the reference simulations are also conducted with the same structures of isotropic materials, i.e. replacing the transformed materials in the triangles BPQ and ABQ with isotropic natural materials of κ_0 =1 W/(m·K).

The temperature profiles of the proposed design and the reference structure are shown in Fig. 3. The required thermal conductivity tensors of the triangles BPQ and ABQ are $\kappa_{BPQ} = \begin{bmatrix} 3.33 & -3 \\ -3 & 3 \end{bmatrix}$ W/(m·K) and $\kappa_{ABQ} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ W/(m·K) respectively according to Eq. (2). Heat is conducted from the incident plane to the exit plane gradually. When comparing the isotherms of the two temperature fields, one may discover that the isotherms in our design are parallel to each other in the left part and in the right part, and the isotherms in the left part are perpendicular to those in right part. While in the reference simulation, the isotherms are curved lines. As we know, the direction of heat flow is perpendicular to the isotherms. Therefore, the direction of heat flow in the left part in the proposed design is along the *x*-axis, while the direction in the right part is along the *y*-axis, as denoted by the red arrow in Fig. 3(a). The direction of heat flow just one-time changes by 90° in our design straightforwardly, just like the light beam being reflected by the mirror. The specific structures in the dash-line box play the role of mirror, which is the exact reason why we call them as thermal reflection meta-device. In contrast, since the isotherms are curved in the reference simulation, the heat flow changes directions gradually along the heat flow lines.

In fact, we not only can change the heat flow direction by 90° by the thermal reflection meta-device, but also can change the heat flow to arbitrary direction almost at will. As shown in Figs. 4(a) and 4(b), when we set the coordinates of point P as $(1+\sqrt{2}, -2+\sqrt{2})$ and $(1+\sqrt{2}, -2-\sqrt{2})$ respectively, we can change the heat flow direction by 45° and 135° through similar method. The required thermal conductivity tensors of the triangle BPQ in Figs. 4(a) and 4(b) are $\kappa_{BPQ} = \begin{bmatrix} 2.24 & -1.18 \\ -1.18 & 1.07 \end{bmatrix}$ W/(m·K) and $\kappa_{BPQ} = \begin{bmatrix} 4.47 & -7.89 \\ -7.89 & 14.13 \end{bmatrix}$ W/(m·K), respectively. Parallel isotherms are observed in the temperature

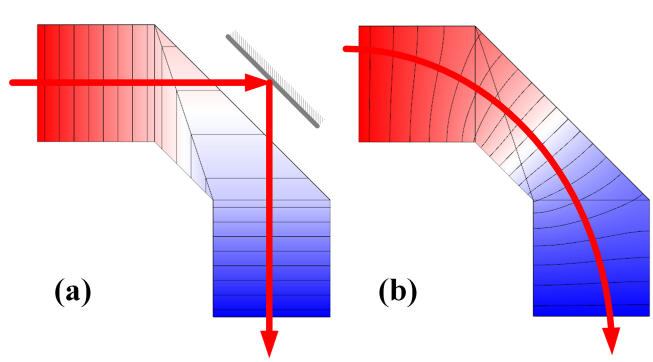

FIG. 3. Simulated temperature profiles of (a) the proposed design and (b) the reference structure. Black lines inside the structures denote the isotherms. Red arrows denote the heat flow direction.

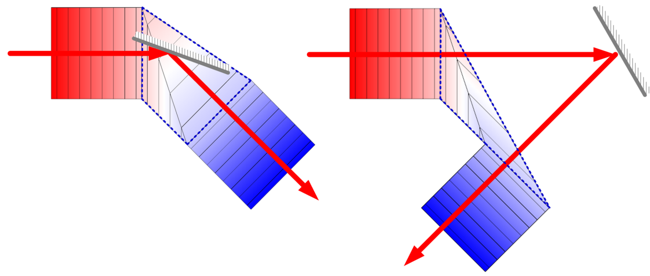

FIG. 4. Heat flow direction changed by (a) 45°, (b) 135° through thermal reflection meta-devices. Black lines inside the structures denote the isotherms. Red arrows denote the heat flow direction.

fields, which indicates that both the directions of heat flow are one-time changed straightforwardly, just like being reflected by an imaginary mirror.

One may ask what benefits we can get when using the thermal reflection meta-device rather than the isotropic materials. To answer this question, we calculate the generalized thermal resistance of above heat transfer processes in the proposed design and the reference structure.[24–26] For the heat conduction process, the generalized thermal resistance can be calculated as

$$R_h = \frac{\Delta T}{Q_{net}} = \frac{\iiint_V \kappa_{ij} \left(\nabla T_j\right)^2 dV}{{Q_{net}}^2} \tag{4}$$

where ΔT is the temperature difference and Q_{net} is the net heat flux through the system. The calculated normalized thermal resistance of the proposed designs and the reference structure in Fig. 4 are shown in Table I. When the heat flow is changed by 45°, the thermal resistance is equivalent; when the heat flow is changed to 90° and 135°, the thermal resistance of the proposed designs becomes smaller than that of the reference structures. When the heat flux is changed by 135°, the thermal resistance is nearly doubled in the reference structure. This can be attributed to the squeezing of heat flux caused by the change of cross-section area, as denoted in Fig. 4. However, the thermal resistance of the proposed designs is almost the same. This characteristic implies that the proposed designs possess lower thermal resistance and higher heat transfer ability.

It should be noted that such thermal reflection meta-device requires some negative thermal conductivity, as seen from the transformed thermal conductivity tensors which is a major restriction for the realization of our designs. Physically, a negative thermal conductivity implies that an external work is required to convey the heat from the low temperature to the high temperature region to comply with the second law of thermodynamics.[27,28] Plentiful papers have report such negative thermal conductivity.[15,28–31] Here, we may see that the required thermal conductivity tensors are spatially invariant and symmetrical, thus we can find a rotation transformation mapping such specific tensors into diagonal ones. Then with the help of effective medium theory (EMT), we can realize the required inhomogeneous distribution of thermal conductivity tensor by two layered isotropic materials A and B.[4,5,32,33] The required thermal conductivity of isotropic materials A and B can be calculated as $\kappa_{A,B} = \chi_1 \pm \sqrt{\chi_1^2 - \chi_1\chi_2}$, where $\chi_{1,2} = \left[\kappa_{11} + \kappa_{22} \pm \sqrt{(\kappa_{11} - \kappa_{22})^2 + 4\kappa_{12}^2}\right]/2$. The rotation angle of the layered structures can be uniquely determined as $\theta = \arctan\left[2\kappa_{12}/(\kappa_{11} - \kappa_{22})\right]/2$.[20,21] One successful demonstration of such thermal reflection meta-device is shown in Fig. 5. As shown in

TABLE I. Comparisons of normalized generalized thermal resistances.

	45°	90°	135°
Proposed designs	0.58 K/W	0.59 K/W	0.52 K/W
Reference structures	0.60 K/W	0.71 K/W	1.00 K/W

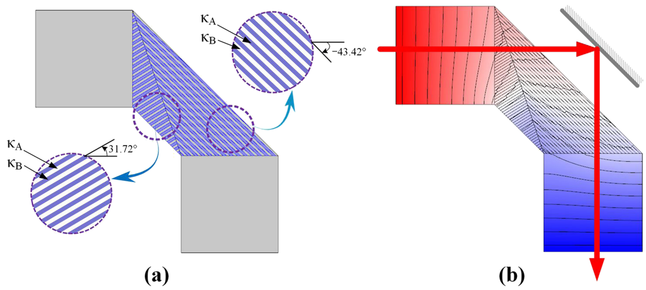

FIG. 5. (a) Schematic of required layered structures. Two isotropic materials A and B are required to fabricate the layered structures with uniquely determined rotation angles. (b) Corresponding temperature field. The heat flux is approximately "reflected" by 90°. Black lines inside the structures denote the isotherms. Red arrows denote the heat flow direction.

Fig. 5(a), the thermal conductivities of materials A and B are 12.26 and 0.08 W/(m·K) in the triangle BPQ with the rotation angle is -43.42°; the thermal conductivities of materials A and B are 5.04 and 0.20 W/(m·K) in the triangle ABQ with the rotation angle is +31.72°. With such required structures made by isotropic nature-existing materials, we also conduct the finite-element simulations, and the result is shown in Fig. 5(b). The temperature fields and isotherm distributions are similar to those in Fig. 3(a) and the heat flux is approximately "reflected" by 90° though the thermal reflection effect is kind of weakened. But the weakened thermal reflection effect could be improved with thinner-layer structures or larger layer number N.[20] It is demonstrated that the present thermal reflection meta-device can also be made by nature-existing isotropic materials. With directional heat transfer, the present meta-device could be used to realize local heating/cooling for electronic devices[34] and high-efficient energy harvesting/utilizing.

IV. CONCLUSIONS

In summary, we design the thermal reflection meta-device to realize the directional heat transport through transformation thermodynamics. Through finite element simulations, we find that the proposed designs can change the heat flux for one-time directly, just like the mirror to the light beam. By calculating the generalized thermal resistance, we verify that such thermal reflection meta-device possesses low thermal resistance and high heat transfer ability. Further, we demonstrate that the proposed thermal reflection meta-device can also be made by nature-existing isotropic materials. Such materials can control/manipulate the heat flow flexibly to realize directional heat transport with least energy loss, which may be quite promising in future applications.

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