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Analysis of elliptical thermal cloak based on entropy generation and entransy dissipation approach*

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In this work, we designed the elliptical thermal cloak based on the transformation thermotics. The local entropy generation rate distribution and entransy dissipation rate distribution were obtained, and the total entropy generation and entransy dissipation of different types of elliptical cloaks were evaluated. We used entropy generation approach and entransy dissipation approach to evaluate the performance of the thermal cloak, and heat dissipation analysis was carried out for models with different parameters. Finally, the optimized elliptical thermal cloak with minimum entropy generation and minimum entransy dissipation is found, and some suggestions on optimizing the structure of elliptical thermal cloak were given.

Keywords: metamaterials, thermal conductivity, entropy

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1. Introduction

With the birth of thermal metmaterials, heat flow manipulation becomes feasible and many new thermal functionalities have been reported, such as thermal cloaks,[1–7] thermal concentrators,[8–10] thermal rotators,[11,12] thermal diodes,[13] thermal illusions,[14–16] thermal camouflages,[17,18] *etc*. The typical tool to design such thermal metamaterial is the transformation thermotics, which is inspired and extended from the seminal transformation optics theory. The essence of the transformation thermotics lies in the invariance of heat conduction equation under coordinate transformation, *i.e.*, $\nabla(\kappa\cdot\nabla T)=0$ in one space changes to $\nabla'(\kappa'\cdot\nabla' T)=0$ in another space, where κ' is the thermal conductivity and T is the temperature. To maintain the same temperature field, the transformed thermal conductivity κ' becomes rather complicated with anisotropy, inhomogeneity, and even singularity. One of the typical functionality of the transformation thermotics is thermal cloak, which can guide heat flux around the certain region and has no influence on the temperature distribution of the background. It seems as if the heat conduction happens on a homogenous plate, and the target objects sitting inside the cloaking region will be concealed from the outside observers. Among other functionalities, thermal cloaking is most attractive and eye-catching because its optical counterpart is a long dream for humans and has been frequently referred in fairy tale, movies, and fictions. Nevertheless, two long-standing challenges remain formidable for thermal cloaks. For one thing, most of the existing thermal cloaks are circular for the easy design and experimental validation. But when extending to the elliptical thermal cloak, the degree of freedom will be enhanced greatly, which can easily unify circular cloaks with different aspect ratios by designing the two axes.[19] While the elliptical cloaks have not been investigated as comprehensive as the circular one. For another thing, most the existing work on thermal cloaks only achieve the realization of the thermal cloaking functionality qualitatively without further discussion on the performance evaluation and optimization quantitatively. Although people use the temperature distribution along the slices or the cut lines to show the effectiveness and success of different thermal functionalities, the overall evaluation is more important after all. For this end, it is very important to find methods to estimate the cloaking performance quantitatively to further guide the structural optimization. In this paper, we introduced entropy generation and entransy dissipation as the two principles to evaluate overall cloaking performances of the elliptical thermal cloak. Different elliptical cloaks were designed and simulated with different parameters, followed by some suggestions to guide the design of the elliptical thermal cloak.

2. Transformation thermotics

A typical elliptical thermal cloak is shown in Fig. 1(a) whose inner and outer semimajor axes along the y direction are a and b respectively. K is the aspect ratio of semimajor

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axes along the y direction and x direction. In order to achieve perfect cloaking performance, we compressed the elliptical cloak's outer boundary ($r = R_2(\theta)$) into the inner ($r' = R_1(\theta)$) boundary along the radical direction. Therefore, the aspect ratio of the transformed ellipse is maintained invariant. When $K > 1$, it represents horizontal elliptical cloaks; when $K < 1$, it represents vertical elliptical cloaks; when $K = 1$, it represents circular cloak. There are three different regions of the thermal cloak as shown in Fig. 1(b): external region (I), cloaking region (II), and core region (III).

To build the relationship of the original and the transformation coordinate spaces, we chose an arbitrary point $M(x,y)$ in the original coordinate space and a corresponding point $M'(x',y')$ in the transformation coordinate space as shown in Fig. 1. The elliptical cloaking coordinate transformation equation can be expressed as:[17]

$$\begin{cases} x' = \left(\dfrac{b-a}{b} + \dfrac{Ka}{\sqrt{x^2+K^2y^2}}\right)x, \\ y' = \left(\dfrac{b-a}{b} + \dfrac{Ka}{\sqrt{x^2+K^2y^2}}\right)y. \end{cases} \tag{1}$$

According to the principle of the transformation thermotics,[18,19] the Fourier equation maintains its original form after coordinate transformation, which can be expressed as:$\rho' c' \partial T'/\partial t = \nabla' \cdot (\boldsymbol{\kappa}' \nabla T')$, where ρ' and c' are the density and thermal capacity, $\boldsymbol{\kappa}'$ is the thermal conductivity tensor, and T' is the temperature. For steady heat conduction, we only consider the thermal conductivity of the cloak in the transformed space, which can be expressed as:

$$\boldsymbol{\kappa}' = \frac{\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}}{\det(\boldsymbol{A})}\kappa = \begin{bmatrix} \kappa_{xx} & \kappa_{xy} \\ \kappa_{yx} & \kappa_{yy} \end{bmatrix} = \begin{bmatrix} \dfrac{\sigma^{1/2}}{\sigma^{1/2}-Ka} + \dfrac{\varsigma - 2Ka\sigma^{2/3}}{(\sigma^{1/2}-Ka)\sigma^{5/2}}x^2 & \dfrac{\varsigma - Ka(1+K^2)\sigma^{2/3}}{(\sigma^{1/2}-Ka)\sigma^{5/2}}xy \\ \dfrac{\varsigma - Ka(1+K^2)\sigma^{2/3}}{(\sigma^{1/2}-Ka)\sigma^{5/2}}xy & \frac{\sigma^{1/2}}{\sigma^{1/2}-Ka} + \frac{\varsigma-2K^3a\sigma^{2/3}}{(\sigma^{1/2}-Ka)\sigma^{5/2}}y^2 \end{bmatrix}\kappa, \tag{2}$$

where A is the Jacobian matrix of the coordinate transformation as $\boldsymbol{A} = \partial(x',y')/\partial(x,y)$, $\sigma = x^2 + K^2y^2$, and $\varsigma = (x^2 + K^4y^2)K^2a^2$.

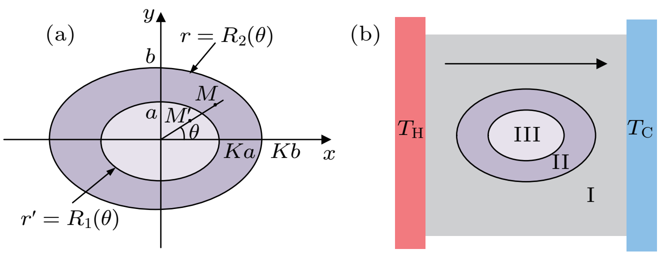

Fig. 1. (a) Sketch of elliptical thermal cloak in the Cartesian coordinate system. (b) Schematic diagram of simulations illustrating that heat is conducted from the left boundary to the right boundary.

3. Overall performance evaluation

The thermal cloak is used to shielding heat from entering the cloaking region and the higher thermal insulating efficiency is, the better cloaking performance will be obtained. On the one hand, entropy generation is a measurement of the chaos of the system.[22,23] When we give the thermodynamic difference, the minimum entropy generation directly correspond to the minimum rate of heat transfer, that is, the best insulating efficiency.[24] On the other hand, entransy represents the heat transfer potential capacity. In the heat transfer process, entransy is dissipative and the minimum entransy dissipation represents the minimum potential capacity dissipation and the optimized heat transform progress.[25,26] Therefore, to evaluate the overall thermal cloaking performance and heat transform progress, we can calculate the entropy generation and entransy dissipation.

In heat conduction, the area entropy generation rate S'' can be written as:[27,28]

$$S'' = \frac{\partial}{\partial x'}\left(\frac{q_x}{T}\right) + \frac{\partial}{\partial y'}\left(\frac{q_y}{T}\right), \tag{3}$$

where q_x and q_y are the heat flow along the x and y directions respectively, and can be calculated as:

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = -\begin{pmatrix} \kappa_{xx} & \kappa_{xy} \\ \kappa_{xy} & \kappa_{yy} \end{pmatrix}\begin{pmatrix} \nabla T_x \\ \nabla T_y \end{pmatrix}. \tag{4}$$

The total entropy generation rate of the cloak can be obtained by integrating the area entropy generation rate:[23,29]

$$S = \iint S''\mathrm{d}A = \iint_{A_{\mathrm{I}}} \dot{S}''_{\mathrm{I}}\mathrm{d}A_{\mathrm{I}} + \iint_{A_{\mathrm{II}}} \dot{S}''_{\mathrm{II}}\mathrm{d}A_{\mathrm{II}} + \iint_{A_{\mathrm{III}}} \dot{S}''_{\mathrm{III}}\mathrm{d}A_{\mathrm{III}}, \tag{5}$$

where $\dot{S}''_{\mathrm{I}}$, $\dot{S}''_{\mathrm{II}}$, $\dot{S}''_{\mathrm{III}}$ are the entropy generation rates of the external region (I), cloaking region (II), and the core region (III), and A_{I}, A_{II}, A_{III} are the area of these three regions.

The area entransy dissipation rate (in units $(\mathrm{W \cdot K})/\mathrm{m}^2$) in heat conduction can be obtained by:[25,30]

$$G'' = \frac{\partial}{\partial x'}(q_x \cdot T) + \frac{\partial}{\partial y'}(q_y \cdot T). \tag{6}$$

The total entransy dissipation rate of the cloak can be calculated by integrating the area entransy dissipation rate:

$$G = \iint G''\mathrm{d}A = \iint_{A_{\mathrm{I}}} \dot{G}''_{\mathrm{I}}\mathrm{d}A_{\mathrm{I}} + \iint_{A_{\mathrm{II}}} \dot{G}''_{\mathrm{II}}\mathrm{d}A_{\mathrm{II}}$$

$$+\iint_{A_{\mathrm{III}}} \dot{G}''_{\mathrm{III}} \mathrm{d}A_{\mathrm{III}}, \qquad (7)$$

where $\dot{G}''_{\mathrm{I}}, \dot{G}''_{\mathrm{II}}, \dot{G}''_{\mathrm{III}}$ are the entropy generation rates of the external region (I), cloaking region (II), and the core region (III).A_{I}, A_{II}, A_{III} are the area of these three regions.

4. Results and discussion

In order to verify the feasibility of the overall evaluation of thermal cloaking performance, COMSOL software is used to calculate the entropy generation and the entransy dissipation for elliptical cloaks with different aspect ratio K. Polydimethylsiloxane (PDMS) is selected to fabricate external region (I) and core region (III) with a thermal conductivity $\kappa = 0.16$ W/(m·K). The thermal conductivity of the cloaking region (II) is obtained according to Eq. (2). The simulation setup is shown in Fig. 1(b). In the simulation, the computational region is a 150 mm×150 mm plate, with a constant high temperature $T_{\mathrm{H}} = 353$ K on the left and a constant low temperature $T_{\mathrm{L}} = 293$ K on the right. The other boundaries are insulated, and the mesh independence is tested before the evaluation process.

4.1. Entropy and entransy analysis of the elliptical thermal cloak

Elliptical thermal cloaks with different aspect ratios K ($K = 0.2$, 0.5, 1, 2, 5) are simulated and analyzed. Figure 2 shows the temperature, entropy generation rate (S'') and entransy dissipation rate (G'') distributions of each cases. It is seen that no heat enters the core region in the thermal cloaks with different K, which implies that any objects sitting in the core region will make no influence to the external temperature distribution. Therefore, the thermal cloaking effect is achieved in these elliptical thermal cloaks, just like the thermal cloaking effect in previous literature. We can also notice that the S'' and G'' have similar distribution under the same K. That is because in our thermal cloak system, the ambient temperature and the heat transfer temperature difference are constant, so S'' and G'' are directly proportional to the heat flux.[26,31] In the core region (III), heat flux is almost zero, so both the S'' and G'' approximate to zero. In the cloaking region (II), due to the anisotropic thermal conductivities, the energy disorder causes the significant fluctuation of the S'' and G''. In the external region (I), there is almost no fluctuation in the S'' and G'' because the thermal temperature gradient of the background is uniform.

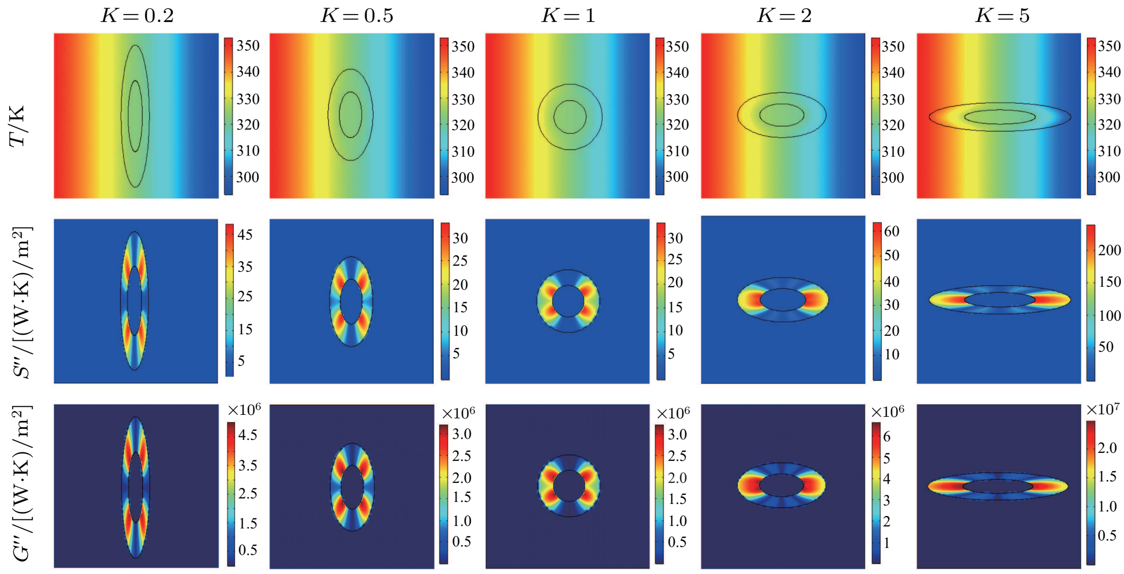

Fig. 2. Temperature, entropy generation rate, and entransy dissipation rate distributions of the elliptical thermal cloaks with different K.

Figure 3 shows the S and G distributions on the symmetry line ($y = 75$ mm) with K of 0.5, and the pure plate with the same boundary conditions was calculated as comparison. For the pure plate, S and G keep constant along the symmetry line. For the thermal cloak, in the core region (III), the heat flux is almost zero thus the S and G are also approach zero, which means the cloaking structure works. In the cloaking region (II), the S and G increase rapidly, which means that stronger thermodynamic processes occurred in this region. We can also see that the S and G on the boundaries between the core region (III) and cloaking region (II) reach the peaks. That is because the boundaries are directly integrated with PDMS and metamaterials, resulting in anisotropic thermal conductivity and the perturbations of thermal energy.[32] Besides, the S and G of the thermal cloak in external region (I) are approximated to that of the pure plate, which indicates that the thermal cloak has little effect on the external space. Moreover, the S and G have similar trend which reveals both entropy generation and entransy dissipation methods can characterize the heat transfer process of the thermal cloak.

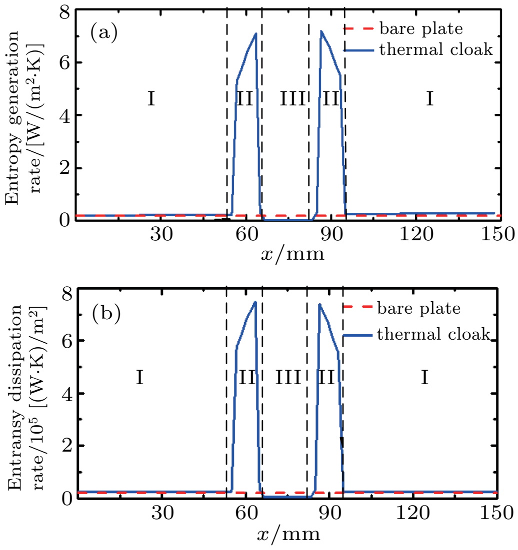

Fig. 3. Entropy generation and entransy dissipation rate distributions on the symmetry line ($y = 75$ mm) of pure plate and thermal cloak with K of 0.5.

4.2. Parametrical optimization of elliptical thermal cloak

As discussed above, the entropy generation and entransy dissipation mainly occur in the cloaking region (II). In order to increase the cloaking performance and reduce the energy losses, we use the entropy generation minimization and entransy dissipation minimization as the two principles to design the thermal cloak.

To investigate the effect the area of cloaking region (II) on its thermal performance, we design five elliptical thermal cloaks with b = 15, 20, 25, 30, 35 mm respectively while maintaining $K = 2$ and $a = 10$ mm. The five cases were simulated and the results are shown in Fig. 4(a) and 4(b). It can be seen that entropy generation rate (S) and entransy generation rate (G) decrease as b increases which means larger cloaking region (II) leads to weaker energy distortion with smaller heat dissipation loss. To investigate the effect of area of the core region (III) on its thermal performance, we design five elliptical thermal cloaks with a = 5, 10, 15, 20, 25 mm respectively while maintaining $K = 2$ and $b = 30$ mm. The simulation results are shown in Figs. 4(c) and 4(d). It can be seen that S and G increase as a increases. Therefore, we can increase the area of the cloaking region (II) appropriately in practical application, but should be coupled with other conditions as well.

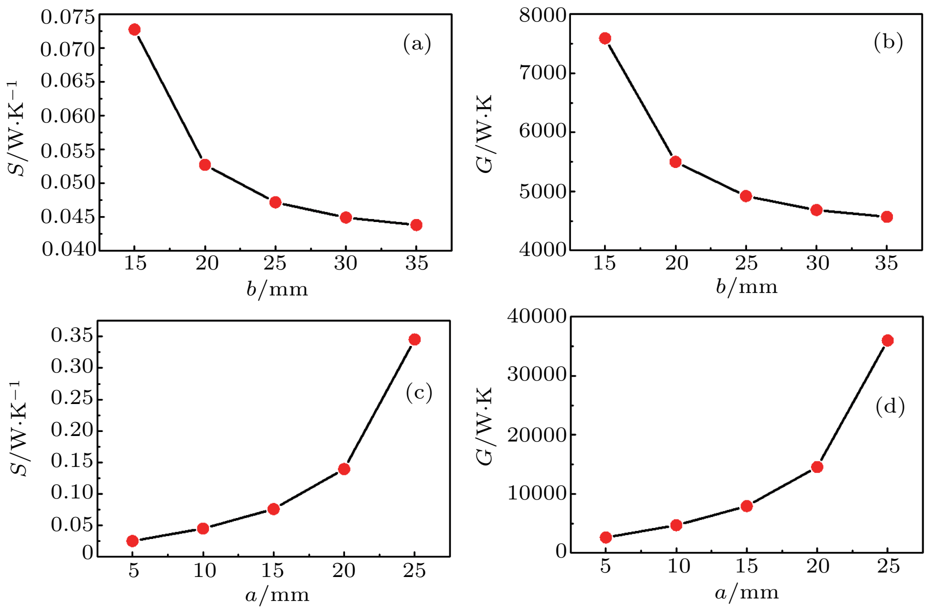

Fig. 4. (a) Relationship between S and b. (b) Relationship between G and b. (c) Relationship between S and a. (d) Relationship between G and a.

As analyzed above, it is found that the entropy generation and entransy dissipation of the elliptical thermal cloaks vary with the change of K. To further analyze how K influences the thermal performance of cloaks, the area of core region and cloaking region are fixed, while K are set as 0.2, 0.5, 1, 2, 3, 4, and 5 respectively. These seven cases were simulated in COMSOL and the results are shown in Fig. 5. We can see that the S and G at $K < 1$ is much smaller than that of $K > 1$, which means that the vertical thermal cloaks are better than the horizontal thermal cloaks. When we put the same elliptical cloak horizontally, the distance of heat transfer along the x direction in the cloaking region is longer than that when we put it vertically. Consequently, this led to more irreversible energy losses and thermal dissipations in the cloaking region,[32] which are reflected by the increasing S and G. Moreover, it shows that for vertical elliptical cloaks ($K < 1$), S and G decrease with K increases. However, for horizontal elliptical cloaks ($K > 1$), S and G increase with K increases. The minimum S and G happens at $K = 1$, which indicates that the rounder the cloak is, the better properties will be obtained. This can be explained as the rounder the cloak is, the distance difference of heat transfer between the x direction and y direction is shorter. So the thermal energy can transfer equally in all directions, resulting in less energy losses and thermal dissipations.

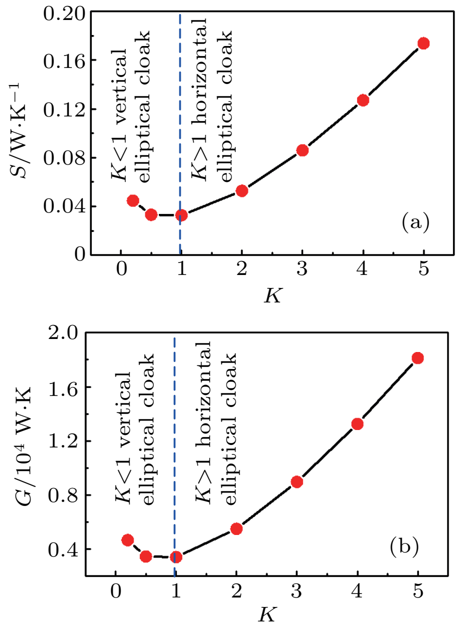

Fig. 5. (a) Relationship between S and K. (b) Relationship between G and K.

5. Conclusion and perspectives

In this paper, we derived the thermal conductivity of the elliptical thermal cloak based on the theory of transformation thermotics. The main goal of the study is to use entropy generation and entransy dissipation as two principles to evaluate the thermal performance of the cloaks. Furthermore, the effect of the design parameters (areas of the core region and cloaking region, axis ratio K) of the cloaks were investigated by numerical simulations. This results show that smaller area of core region and larger the area of cloaking region lead to better thermal performance of the elliptical thermal cloaks. The vertical thermal cloaks with $K<1$ have better thermal performance than the horizontal ones with $K>1$, and the circular cloak has the best performance. This work contributes to optimizing the elliptical thermal cloak by providing entropy generation and entransy dissipation as principles. More broadly, the present overall evaluation method can also be extended to evaluate and design other thermal functionalities.

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